

The Research-  
Driven Solution  
to Raise the  
Quality of High  
School Core  
Courses

**QualityCore<sup>®</sup>**



# Algebra II

**ACT Course Standards**



## ACT Course Standards Algebra II

A set of empirically derived course standards is the heart of each QualityCore® mathematics course. The ACT Course Standards represent a solid evidence-based foundation in mathematics. They were developed from an intensive study of high-performing high schools with significant minority and low-income enrollments that produced many graduates who met or exceeded ACT College Readiness Benchmark Scores (See <http://www.act.org/path/policy/reports/success.html>).

This document contains a list of ACT Course Standards for a rigorous Algebra II course—what students should know and be able to do in the course—and a worksheet teachers can use to compare their course content to these standards. The ACT standards encompass the following overarching themes and/or foundational concepts:

- A. Prerequisites
- B. Exploring the Skills and Strategies Underlying Mathematics
- C. Establishing Number Sense and Operation Skills
- D. Exploring Expressions, Equations, and Functions in the First Degree
- E. Exploring Quadratic Equations and Functions
- F. Exploring Polynomial Expressions, Equations, and Functions
- G. Exploring Advanced Functions
- H. Organizing and Analyzing Data and Applying Probability
- I. Using Matrices to Organize Data and Solve Problems

### ACT Course Standards—Algebra II

A. Prerequisites	
1. Skills Acquired by Students in a Previous Course and Refined in This Course	
a.	Identify properties of real numbers and use them and the correct order of operations to simplify expressions
b.	Multiply monomials and binomials
c.	Factor trinomials in the form $ax^2 + bx + c$
d.	Solve single-step and multistep equations and inequalities in one variable
e.	Solve systems of two linear equations using various methods, including elimination, substitution, and graphing
f.	Write linear equations in standard form and slope-intercept form when given two points, a point and the slope, or the graph of the equation
g.	Graph a linear equation using a table of values, $x$ - and $y$ -intercepts, or slope-intercept form
h.	Find the distance and midpoint between two points in the coordinate plane
i.	Use sine, cosine, and tangent ratios to find the sides or angles of right triangles
j.	Use inductive reasoning to make conjectures and deductive reasoning to arrive at valid conclusions

**B. Exploring the Skills and Strategies Underlying Mathematics****1. Mathematical Processes Learned in the Context of Increasingly Complex Mathematical and Real-World Problems**

(Note: These mathematical processes are the same for Algebra I, Geometry, Algebra II, and Precalculus.)

- |    |  |
|----|--|
| a. | Apply problem-solving skills (e.g., identifying irrelevant or missing information, making conjectures, extracting mathematical meaning, recognizing and performing multiple steps when needed, verifying results in the context of the problem) to the solution of real-world problems |
| b. | Use a variety of strategies to set up and solve increasingly complex problems  |
| c. | Represent data, real-world situations, and solutions in increasingly complex contexts (e.g., expressions, formulas, tables, charts, graphs, relations, functions) and understand the relationships   |
| d. | Use the language of mathematics to communicate increasingly complex ideas orally and in writing, using symbols and notations correctly   |
| e. | Make appropriate use of estimation and mental mathematics in computations and to determine the reasonableness of solutions to increasingly complex problems  |
| f. | Make mathematical connections among concepts, across disciplines, and in everyday experiences  |
| g. | Demonstrate the appropriate role of technology (e.g., calculators, software programs) in mathematics (e.g., organize data, develop concepts, explore relationships, decrease time spent on computations after a skill has been established)  |
| h. | Apply previously learned algebraic and geometric concepts to more advanced problems  |

**C. Establishing Number Sense and Operation Skills****1. Foundations**

- |    |   |
|----|---|
| a. | Identify complex numbers and write their conjugates   |
| b. | Add, subtract, and multiply complex numbers   |
| c. | Simplify quotients of complex numbers   |
| d. | Perform operations on functions, including function composition, and determine domain and range for each of the given functions |

**D. Exploring Expressions, Equations, and Functions in the First Degree****1. Expressions, Equations, and Inequalities**

- |    |  |
|----|--|
| a. | Solve linear inequalities containing absolute value                              |
| b. | Solve compound inequalities containing “and” and “or” and graph the solution set |
| c. | Solve algebraically a system containing three variables                          |

**2. Graphs, Relations, and Functions**

- |    |  |
|----|--|
| a. | Graph a system of linear inequalities in two variables with and without technology to find the solution set to the system          |
| b. | Solve linear programming problems by finding maximum and minimum values of a function over a region defined by linear inequalities |

**E. Exploring Quadratic Equations and Functions****1. Equations and Inequalities**

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|----|--|
| a. | Solve quadratic equations and inequalities using various techniques, including completing the square and using the quadratic formula |
| b. | Use the discriminant to determine the number and type of roots for a given quadratic equation  |
| c. | Solve quadratic equations with complex number solutions  |
| d. | Solve quadratic systems graphically and algebraically with and without technology  |

**2. Graphs, Relations, and Functions**

- |    |  |
|----|--|
| a. | Determine the domain and range of a quadratic function; graph the function with and without technology                         |
| b. | Use transformations (e.g., translation, reflection) to draw the graph of a relation and determine a relation that fits a graph |
| c. | Graph a system of quadratic inequalities with and without technology to find the solution set to the system                    |

**3. Conic Sections**

- |    |  |
|----|--|
| a. | Identify conic sections (e.g., parabola, circle, ellipse, hyperbola) from their equations in standard form             |
| b. | Graph circles and parabolas and their translations from given equations or characteristics with and without technology |

c.	Determine characteristics of circles and parabolas from their equations and graphs
d.	Identify and write equations for circles and parabolas from given characteristics and graphs
<b>F. Exploring Polynomial Expressions, Equations, and Functions</b>	
<b>1. Expressions and Equations</b>	
a.	Evaluate and simplify polynomial expressions and equations
b.	Factor polynomials using a variety of methods (e.g., factor theorem, synthetic division, long division, sums and differences of cubes, grouping)
<b>2. Functions</b>	
a.	Determine the number and type of rational zeros for a polynomial function
b.	Find all rational zeros of a polynomial function
c.	Recognize the connection among zeros of a polynomial function, x-intercepts, factors of polynomials, and solutions of polynomial equations
d.	Use technology to graph a polynomial function and approximate the zeros, minimum, and maximum; determine domain and range of the polynomial function
<b>G. Exploring Advanced Functions</b>	
<b>1. Rational and Radical Expressions, Equations, and Functions</b>	
a.	Solve mathematical and real-world rational equation problems (e.g., work or rate problems)
b.	Simplify radicals that have various indices
c.	Use properties of roots and rational exponents to evaluate and simplify expressions
d.	Add, subtract, multiply, and divide expressions containing radicals
e.	Rationalize denominators containing radicals and find the simplest common denominator
f.	Evaluate expressions and solve equations containing $n$ th roots or rational exponents
g.	Evaluate and solve radical equations given a formula for a real-world situation
<b>2. Exponential and Logarithmic Functions</b>	
a.	Graph exponential and logarithmic functions with and without technology
b.	Convert exponential equations to logarithmic form and logarithmic equations to exponential form
<b>3. Trigonometric and Periodic Functions</b>	
a.	Use the law of cosines and the law of sines to find the lengths of sides and measures of angles of triangles in mathematical and real-world problems
b.	Use the unit-circle definition of the trigonometric functions and trigonometric relationships to find trigonometric values for general angles
c.	Measure angles in standard position using degree or radian measure and convert a measure from one unit to the other
d.	Graph the sine and cosine functions with and without technology
e.	Determine the domain and range of the sine and cosine functions, given a graph
f.	Find the period and amplitude of the sine and cosine functions, given a graph
g.	Use sine, cosine, and tangent functions, including their domains and ranges, periodic nature, and graphs, to interpret and analyze relations
<b>H. Organizing and Analyzing Data and Applying Probability</b>	
<b>1. Data Relations, Probability, and Statistics</b>	
a.	Use the fundamental counting principle to count the number of ways an event can happen
b.	Use counting techniques, like combinations and permutations, to solve problems (e.g., to calculate probabilities)
c.	Find the probability of mutually exclusive and nonmutually exclusive events
d.	Find the probability of independent and dependent events
e.	Use unions, intersections, and complements to find probabilities
f.	Solve problems involving conditional probability

<b>2. Sequences and Series</b>	
a.	Find the $n$ th term of an arithmetic or geometric sequence
b.	Find the position of a given term of an arithmetic or geometric sequence
c.	Find sums of a finite arithmetic or geometric series
d.	Use sequences and series to solve real-world problems
e.	Use sigma notation to express sums
<b>I. Using Matrices to Organize Data and Solve Problems</b>	
<b>1. Matrices</b>	
a.	Add, subtract, and multiply matrices
b.	Use addition, subtraction, and multiplication of matrices to solve real-world problems
c.	Calculate the determinant of $2 \times 2$ and $3 \times 3$ matrices
d.	Find the inverse of a $2 \times 2$ matrix
e.	Solve systems of equations by using inverses of matrices and determinants
f.	Use technology to perform operations on matrices, find determinants, and find inverses

## ACT Course Standards Worksheet—Algebra II

This worksheet gives teachers an opportunity to compare their course content to ACT's QualityCore® program. Completing the worksheet also allows teachers who teach the same course to ensure their courses have similar outcomes.

### Gap Analysis 1—Individual Teacher Review

This analysis allows individual teachers to identify “gaps” between ACT Course Standards and their course content. They should review the ACT standards on the following worksheet, then determine whether the ACT standard **is** or **is not** included in the course as it is currently taught. “Included” means the standard is taught and students are expected to demonstrate proficiency by the end of the course. “Not Included” means the standard is not taught in the course, is taught in another course, or is already mastered. In the “Gap 1” column on the worksheet, place an “I” for “Included” or an “NI” for “Not Included.” Analyze any gaps between the current course standards and the ACT Course Standards. Identify reasons the standards receiving a “Not Included” designation are not included in the course.

### Gap Analysis 2—Group Consensus

This analysis allows groups of teachers who teach the same course and who have completed Gap Analysis 1 individually to identify differences in how they evaluated the gaps between ACT Course Standards and current course standards. In the “Gap 2” column of the worksheet, place an “X” where members of the group differed in their assessment of whether a particular ACT standard is included in the course as it is currently taught.

The following questions can guide discussion of the gaps:

#### Overarching Questions

1. What should students know and be able to do before going to the next course?
2. Do all teachers teaching this course have a shared understanding of the intent or meaning of each course standard and topic area?

#### Gap Analysis 1 Questions

1. Which ACT Course Standards were identified as not included in the course?
2. What is the level of agreement among the group of teachers about the skills and knowledge that is or is not taught in the course?
3. Are there sound pedagogical reasons for not including specific ACT standards in the course?
4. What implications will any decisions have on students' future learning and academic achievement?

#### Gap Analysis 2 Questions

1. Which of the ACT Course Standards elicited differences of opinion?
2. What are the possible reasons for different opinions about the standards that are or are not included in the course?
3. Are there sound pedagogical reasons for including or not including these disputed standards in the course?
4. What implications will any decisions have on students' future learning and academic achievement?

Finally, document the necessary steps to address the outcomes of the discussion. Be sure to note whether course standards will be added, deleted, or modified; identify who will be responsible for communicating any changes to other teachers; and note any other decisions. Document responsibilities and establish a timetable for continuing the discussion and implementing the decisions.

NOTE: This course content review is most effective as a continuous process that generates feedback throughout the year. ACT recommends, at minimum, monthly status update meetings for teachers and departments involved in the review.

Algebra II Course Standards	Gap 1	Gap 2	Comments
<b>A. Prerequisites</b>			
<b>1. Skills Acquired by Students in a Previous Course and Refined in This Course</b>			
a. Identify properties of real numbers and use them and the correct order of operations to simplify expressions			
b. Multiply monomials and binomials			
c. Factor trinomials in the form $ax^2 + bx + c$			
d. Solve single-step and multistep equations and inequalities in one variable			
e. Solve systems of two linear equations using various methods, including elimination, substitution, and graphing			
f. Write linear equations in standard form and slope-intercept form when given two points, a point and the slope, or the graph of the equation			
g. Graph a linear equation using a table of values, x- and y-intercepts, or slope-intercept form			
h. Find the distance and midpoint between two points in the coordinate plane			
i. Use sine, cosine, and tangent ratios to find the sides or angles of right triangles			
j. Use inductive reasoning to make conjectures and deductive reasoning to arrive at valid conclusions			

Algebra II Course Standards	Gap 1	Gap 2	Comments
<b>B. Exploring the Skills and Strategies Underlying Mathematics</b>			
<b>1. Mathematical Processes Learned in the Context of Increasingly Complex Mathematical and Real-World Problems</b> (Note: These mathematical processes are the same for Algebra I, Geometry, Algebra II, and Precalculus.)			
a. Apply problem-solving skills (e.g., identifying irrelevant or missing information, making conjectures, extracting mathematical meaning, recognizing and performing multiple steps when needed, verifying results in the context of the problem) to the solution of real-world problems			
b. Use a variety of strategies to set up and solve increasingly complex problems			
c. Represent data, real-world situations, and solutions in increasingly complex contexts (e.g., expressions, formulas, tables, charts, graphs, relations, functions) and understand the relationships			
d. Use the language of mathematics to communicate increasingly complex ideas orally and in writing, using symbols and notations correctly			
e. Make appropriate use of estimation and mental mathematics in computations and to determine the reasonableness of solutions to increasingly complex problems			
f. Make mathematical connections among concepts, across disciplines, and in everyday experiences			
g. Demonstrate the appropriate role of technology (e.g., calculators, software programs) in mathematics (e.g., organize data, develop concepts, explore relationships, decrease time spent on computations after a skill has been established)			
h. Apply previously learned algebraic and geometric concepts to more advanced problems			
<b>C. Establishing Number Sense and Operation Skills</b>			
<b>1. Foundations</b>			
a. Identify complex numbers and write their conjugates			
b. Add, subtract, and multiply complex numbers			

<b>Algebra II Course Standards</b>	<b>Gap 1</b>	<b>Gap 2</b>	<b>Comments</b>
c. Simplify quotients of complex numbers			
d. Perform operations on functions, including function composition, and determine domain and range for each of the given functions			
<b>D. Exploring Expressions, Equations, and Functions in the First Degree</b>			
<b>1. Expressions, Equations, and Inequalities</b>			
a. Solve linear inequalities containing absolute value			
b. Solve compound inequalities containing “and” and “or” and graph the solution set			
c. Solve algebraically a system containing three variables			
<b>2. Graphs, Relations, and Functions</b>			
a. Graph a system of linear inequalities in two variables with and without technology to find the solution set to the system			
b. Solve linear programming problems by finding maximum and minimum values of a function over a region defined by linear inequalities			
<b>E. Exploring Quadratic Equations and Functions</b>			
<b>1. Equations and Inequalities</b>			
a. Solve quadratic equations and inequalities using various techniques, including completing the square and using the quadratic formula			
b. Use the discriminant to determine the number and type of roots for a given quadratic equation			
c. Solve quadratic equations with complex number solutions			
d. Solve quadratic systems graphically and algebraically with and without technology			

Algebra II Course Standards	Gap 1	Gap 2	Comments
<b>2. Graphs, Relations, and Functions</b>			
a. Determine the domain and range of a quadratic function; graph the function with and without technology			
b. Use transformations (e.g., translation, reflection) to draw the graph of a relation and determine a relation that fits a graph			
c. Graph a system of quadratic inequalities with and without technology to find the solution set to the system			
<b>3. Conic Sections</b>			
a. Identify conic sections (e.g., parabola, circle, ellipse, hyperbola) from their equations in standard form			
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<b>1. Expressions and Equations</b>			
a. Evaluate and simplify polynomial expressions and equations			
b. Factor polynomials using a variety of methods (e.g., factor theorem, synthetic division, long division, sums and differences of cubes, grouping)			
<b>2. Functions</b>			
a. Determine the number and type of rational zeros for a polynomial function			
b. Find all rational zeros of a polynomial function			

Algebra II Course Standards	Gap 1	Gap 2	Comments
c. Recognize the connection among zeros of a polynomial function, x-intercepts, factors of polynomials, and solutions of polynomial equations			
d. Use technology to graph a polynomial function and approximate the zeros, minimum, and maximum; determine domain and range of the polynomial function			
<b>G. Exploring Advanced Functions</b>			
<b>1. Rational and Radical Expressions, Equations, and Functions</b>			
a. Solve mathematical and real-world rational equation problems (e.g., work or rate problems)			
b. Simplify radicals that have various indices			
c. Use properties of roots and rational exponents to evaluate and simplify expressions			
d. Add, subtract, multiply, and divide expressions containing radicals			
e. Rationalize denominators containing radicals and find the simplest common denominator			
f. Evaluate expressions and solve equations containing $n$ th roots or rational exponents			
g. Evaluate and solve radical equations given a formula for a real-world situation			
<b>2. Exponential and Logarithmic Functions</b>			
a. Graph exponential and logarithmic functions with and without technology			
b. Convert exponential equations to logarithmic form and logarithmic equations to exponential form			

Algebra II Course Standards	Gap 1	Gap 2	Comments
<b>3. Trigonometric and Periodic Functions</b>			
a. Use the law of cosines and the law of sines to find the lengths of sides and measures of angles of triangles in mathematical and real-world problems			
b. Use the unit-circle definition of the trigonometric functions and trigonometric relationships to find trigonometric values for general angles			
c. Measure angles in standard position using degree or radian measure and convert a measure from one unit to the other			
d. Graph the sine and cosine functions with and without technology			
e. Determine the domain and range of the sine and cosine functions, given a graph			
f. Find the period and amplitude of the sine and cosine functions, given a graph			
g. Use sine, cosine, and tangent functions, including their domains and ranges, periodic nature, and graphs, to interpret and analyze relations			
<b>H. Organizing and Analyzing Data and Applying Probability</b>			
<b>1. Data Relations, Probability, and Statistics</b>			
a. Use the fundamental counting principle to count the number of ways an event can happen			
b. Use counting techniques, like combinations and permutations, to solve problems (e.g., to calculate probabilities)			
c. Find the probability of mutually exclusive and nonmutually exclusive events			
d. Find the probability of independent and dependent events			
e. Use unions, intersections, and complements to find probabilities			

Algebra II Course Standards	Gap 1	Gap 2	Comments
f. Solve problems involving conditional probability			
<b>2. Sequences and Series</b>			
a. Find the $n$ th term of an arithmetic or geometric sequence			
b. Find the position of a given term of an arithmetic or geometric sequence			
c. Find sums of a finite arithmetic or geometric series			
d. Use sequences and series to solve real-world problems			
e. Use sigma notation to express sums			
<b>I. Using Matrices to Organize Data and Solve Problems</b>			
<b>1. Matrices</b>			
a. Add, subtract, and multiply matrices			
b. Use addition, subtraction, and multiplication of matrices to solve real-world problems			
c. Calculate the determinant of $2 \times 2$ and $3 \times 3$ matrices			
d. Find the inverse of a $2 \times 2$ matrix			
e. Solve systems of equations by using inverses of matrices and determinants			
f. Use technology to perform operations on matrices, find determinants, and find inverses			