

Tips for Using the QualityCore® Mathematics Benchmark Assessments

Each QualityCore® course has its own set of Benchmark Assessments based on the QualityCore Formative Item Pool. Algebra I has four Benchmark Assessments and Algebra II, Geometry, and Precalculus each have five Benchmark Assessments. Each assessment consists of 15 to 25 multiple-choice items and one constructed-response item.

The assessments are presented as a PDF file to maintain the visual consistency of graphics, special characters, and symbols. Each assessment is “bookmarked” for easy navigation through the PDF file. The PDF file also contains the corresponding QualityCore Reference Sheet.

Each Benchmark Assessment is introduced by a cover sheet displaying the item Identification Number (ID), the correct answer (Key), the cognitive level, and the alphanumeric code for each ACT Course Standard covered by that item. (See the applicable *ACT Course Standards* document.) The scoring criteria and a scoring rubric follow the constructed-response item.

Reference Sheet for the QualityCore™ Algebra II End-of-Course Assessment

Equations of a Line

| | | |
|----------------------|------------------------|--|
| Standard Form | $Ax + By = C$ | A , B , and C are constants with A and B not both equal to zero. |
| Slope-Intercept Form | $y = mx + b$ | (x_1, y_1) is a point. |
| Point-Slope Form | $y - y_1 = m(x - x_1)$ | m = slope b = y-intercept |

Quadratics

| | | |
|---------------------------------------|--|---|
| Standard Form of a Quadratic Equation | $ax^2 + bx + c = 0$ | a , b , and c are constants, where $a \neq 0$. |
| Quadratic Formula | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | |

Conic Sections

| | | |
|-----------|---|--|
| Circle | $(x - h)^2 + (y - k)^2 = r^2$ | center (h, k) r = radius |
| Parabola | $y = a(x - h)^2 + k$ | axis of symmetry $x = h$ vertex (h, k) directrix $y = k - \frac{1}{4a}$ focus $(h, k + \frac{1}{4a})$ |
| Parabola | $x = a(y - k)^2 + h$ | axis of symmetry $y = k$ vertex (h, k) directrix $x = h - \frac{1}{4a}$ focus $(h + \frac{1}{4a}, k)$ |
| Ellipse | $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ | foci $(h \pm c, k)$ where $c^2 = a^2 - b^2$, center (h, k) |
| Ellipse | $\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$ | foci $(h, k \pm c)$ where $c^2 = a^2 - b^2$, center (h, k) |
| Hyperbola | $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ | foci $(h \pm c, k)$ where $c^2 = a^2 + b^2$, center (h, k) |
| Hyperbola | $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$ | foci $(h, k \pm c)$ where $c^2 = a^2 + b^2$, center (h, k) |

Lines and Points

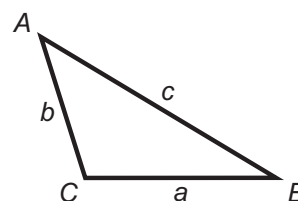
| | | |
|----------|---|--|
| Slope | $m = \frac{y_2 - y_1}{x_2 - x_1}$ | (x_1, y_1) and (x_2, y_2) are 2 points. m = slope |
| Midpoint | $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ | M = midpoint d = distance |
| Distance | $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | |

Miscellaneous

| | | |
|----------------------|--|--|
| Distance, Rate, Time | $D = rt$ | $D =$ distance $r =$ rate $t =$ time |
| Simple Interest | $I = prt$ | $I =$ interest $p =$ principal |
| Compound Interest | $A = p\left(1 + \frac{r}{n}\right)^{nt}$ | $A =$ amount of money after t years $n =$ number of times interest is compounded annually |
| Pythagorean Theorem | $a^2 + b^2 = c^2$ | a and $b =$ legs of right triangle $c =$ hypotenuse |

Laws of Sines and Cosines

| | |
|----------------|--|
| Law of Sines | $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ |
| Law of Cosines | $a^2 = b^2 + c^2 - 2bc \cos A$ |



Sequences, Series, and Counting

| | | |
|---------------------|--|--|
| Arithmetic Sequence | $a_n = a_1 + (n - 1)d$ | $a_n =$ n^{th} term |
| Arithmetic Series | $s_n = \frac{n}{2}(a_1 + a_n)$ | $n =$ number of the term $d =$ common difference |
| Geometric Sequence | $a_n = a_1(r^{n-1})$ | $s_n =$ sum of the first n terms $r =$ common ratio |
| Geometric Series | $s_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$ | $k =$ number of objects in the set $m =$ number of objects selected |
| Combinations | ${}_k C_m = C(k, m) = \frac{k!}{(k-m)! m!}$ | |
| Permutations | ${}_k P_m = P(k, m) = \frac{k!}{(k-m)!}$ | |

Circumference, Area, and Volume

| | | |
|-------------------------|-------------------------------|--|
| Triangle | $A = \frac{1}{2}bh$ | $A =$ area $b =$ base $h =$ height |
| Parallelogram | $A = bh$ | $r =$ radius |
| Trapezoid | $A = \frac{1}{2}(b_1 + b_2)h$ | $C =$ circumference $d =$ diameter |
| Circle | $A = \pi r^2$ $C = \pi d$ | $V =$ volume |
| General Prism | $V = Bh$ | $B =$ area of base $\pi \approx 3.14$ |
| Right Circular Cylinder | $V = \pi r^2 h$ | |
| Pyramid | $V = \frac{1}{3}Bh$ | |
| Right Circular Cone | $V = \frac{1}{3}\pi r^2 h$ | |
| Sphere | $V = \frac{4}{3}\pi r^3$ | |

QualityCore® Benchmark Assessment

Algebra II – Benchmark 1 Linear Functions

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| ID | Key | Cognitive Level | Standard |
|-------|-----|-----------------|----------------|
| 00280 | A | L1 | D.1.a |
| 00281 | A | L1 | D.1.a |
| 00301 | C | L2 | D.1.a |
| 00432 | A | L2 | D.1.b |
| 00569 | C | L2 | D.1.b |
| 00570 | B | L2 | D.1.c |
| 00339 | B | L2 | D.2.a |
| 00534 | A | L2 | D.1.c |
| 00535 | D | L3 | D.2.a |
| 00302 | B | L3 | D.1.b |
| 00284 | B | L3 | D.1.c |
| 00266 | D | L3 | D.2.b |
| 00434 | C | L3 | D.2.b |
| 00571 | D | L3 | D.2.b |
| 00265 | B | L3 | D.2.a |
| 01000 | - | L3 | D.1.a D.1.b |

Directions:

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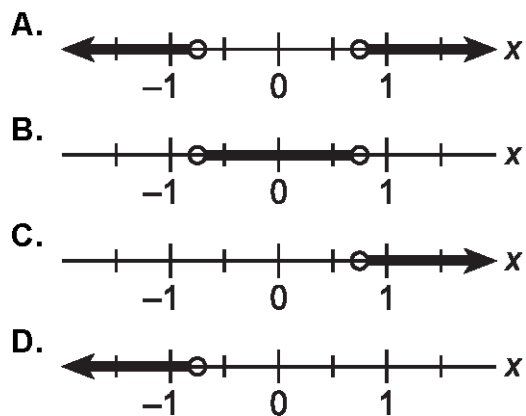
Note: Unless otherwise stated, all of the following assumptions apply to these problems.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word *line* indicates a straight line.
4. The word *average* indicates the arithmetic mean.

Name:
Teacher:

Date:
Class/Period:

1) Which graph represents the solution set of $4|x| - 3 > 0$?



2) What is the solution set to the inequality $|3x - 2| < 7$?

- A. $\{x \mid -\frac{5}{3} < x < 3\}$
- B. $\{x \mid -3 < x < \frac{5}{3}\}$
- C. $\{x \mid x > 3 \text{ or } x < -\frac{5}{3}\}$
- D. $\{x \mid x > \frac{5}{3} \text{ or } x < -3\}$

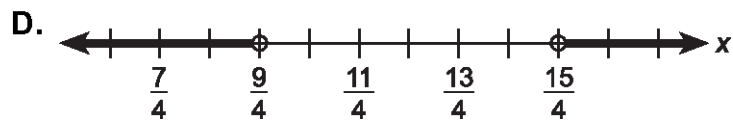
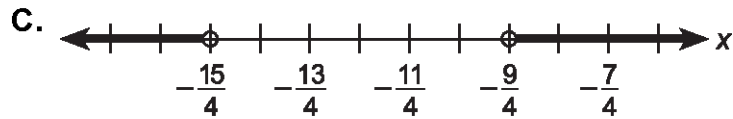
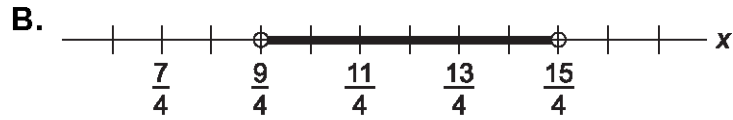
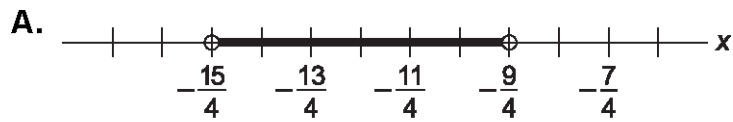
3) Which inequality represents the set of numbers shown by this number line?



- A. $|x + 1| < 8$
- B. $|x - 1| < 8$
- C. $|x + 1| < 4$
- D. $|x - 1| < 4$

4) Which number line shows the solution for

$$|x + 3| < \frac{3}{4} ?$$



5) What is the solution set to the compound inequality $x^2 > 3$ and $x < 2$?

- A. $\{x \mid 2 < x < \sqrt{3}\}$
- B. $\{x \mid \sqrt{3} < x < 2\}$
- C. $\{x \mid x < -\sqrt{3} \text{ or } \sqrt{3} < x < 2\}$
- D. $\{x \mid -\sqrt{3} < x \text{ and } x < \sqrt{3}\}$

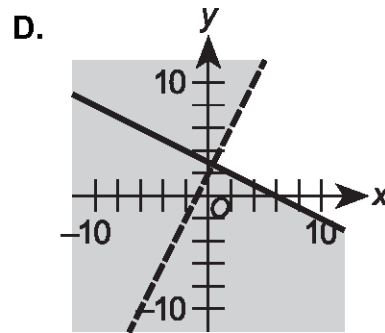
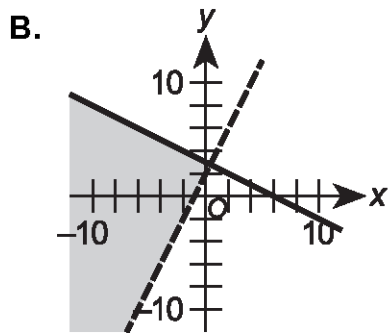
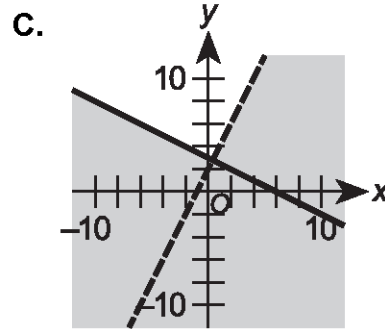
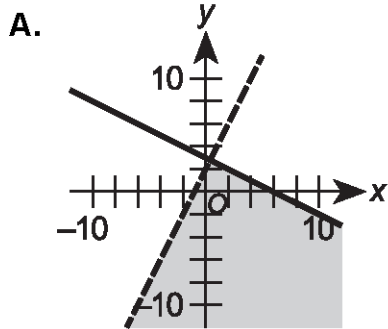
6) What is the value of y in the solution to this system of equations?

$$\begin{cases} 2x + y = 0 \\ 3x - z = 4 \\ x - y + z = 2 \end{cases}$$

- A. -6
- B. -2
- C. 1
- D. 3

7) Which graph represents the solution set of this system of inequalities?

$$\begin{cases} x + 2y \leq 6 \\ 2x - y < -2 \end{cases}$$



8) The graph of a quadratic function passes through the points $(-1, 5)$, $(1, 11)$, and $(3, 1)$. Which system of linear equations in terms of a , b , and c can be used to find the function?

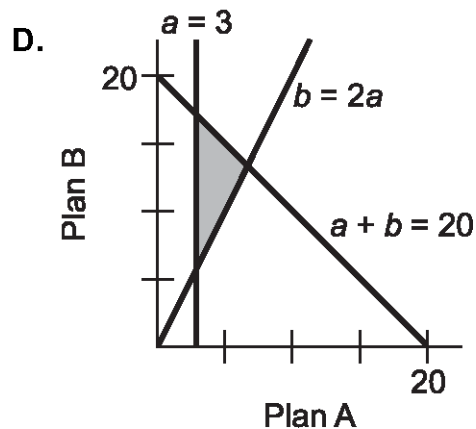
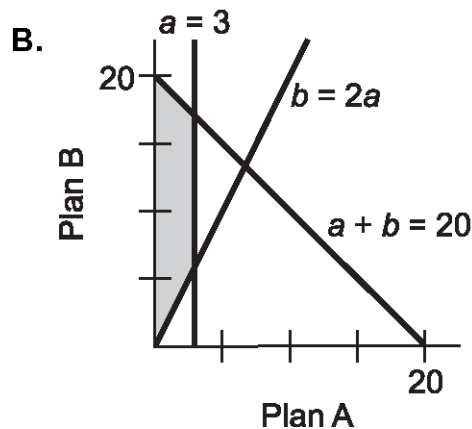
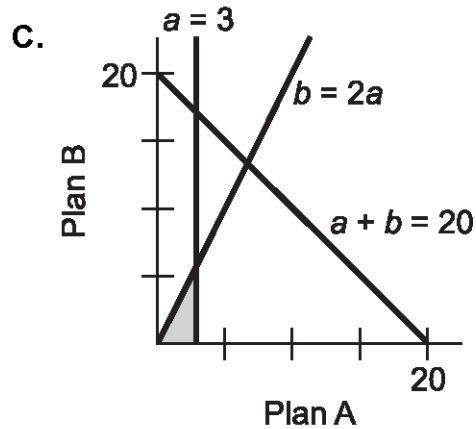
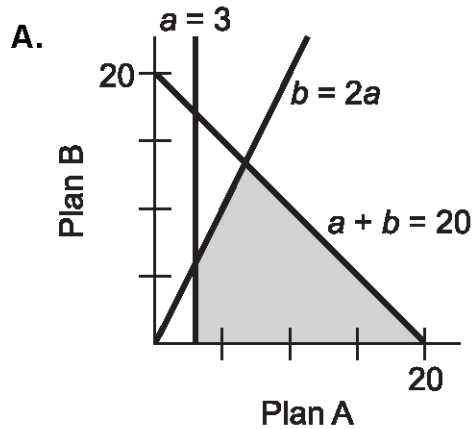
A.
$$\begin{cases} a - b + c = 5 \\ a + b + c = 11 \\ 9a + 3b + c = 1 \end{cases}$$

C.
$$\begin{cases} a - b + c = 5 \\ a + b + c = 11 \\ 3a + 3b + c = 1 \end{cases}$$

B.
$$\begin{cases} a + b + 9c = 5 \\ -a + b + 3c = 11 \\ a + b + c = 1 \end{cases}$$

D.
$$\begin{cases} -a + b + 3c = 5 \\ -a + b + 3c = 11 \\ a + b + c = 1 \end{cases}$$

9) Hyun has at most \$20,000 to invest. She wants to invest at least \$3,000 in Plan A and the rest in Plan B. Her Plan B investment must be at least twice as large as her Plan A investment. Which graph, with axes in terms of thousands of dollars, represents all possibilities for her two investments?



10) Kickball games last a maximum of 60 minutes and ties are allowed. However, there is a “mercy” rule. A game will end if at least 40 minutes have passed and one team is ahead by at least 10 points. If t is time, in minutes, and d is the difference in points, what compound inequality describes the “mercy” rule?

- A. $t \geq 40$ and $d \geq 10$
- B. $40 \leq t < 60$ and $d \geq 10$
- C. $t \leq 40$ or $d \geq 10$
- D. $40 \leq t < 60$ or $d \geq 10$

11) Sally observes that the data derived from an experiment seems to be parabolic when plotted on ordinary graph paper. Three of the observed points are (1,20), (2,21), and (3,18). Use the equation of the parabola that contains these 3 points to determine the y-value at $x = 4$.

- A. $y = -8$
- B. $y = 11$
- C. $y = 17$
- D. $y = 27$

12) Hunter's Transport Company has 7 dump trucks, 5 cement trucks, and 9 drivers. Dump trucks haul 6 tons, while cement trucks haul 10 tons. The company has a contract to transport 360 tons of gravel and cement per day to a road construction site. The dump trucks can make 8 trips a day, while the cement trucks can make 6 trips a day. A dump truck costs \$30 per day, and a cement truck costs \$42 per day. If all 9 drivers work on this job, using how many trucks of each type will minimize the cost?

- A. 4 dump and 5 cement trucks
- B. 5 dump and 4 cement trucks
- C. 6 dump and 3 cement trucks
- D. 7 dump and 2 cement trucks

13) What is the area of the figure determined by this system of inequalities?

$$\begin{cases} -4 \leq x \leq 0 \\ -6 \leq y \leq 0 \end{cases}$$

- A. 6
- B. 15
- C. 24
- D. 28

14) Given:

$$x \geq 0, y \geq 0$$

$$4x + y \geq 12$$

$$4x + 3y \leq 24$$

What is the maximum value of the function
 $f(x,y) = 2x + 3y$?

A. 6

B. 12

C. 17

D. 21

15) Given:

$$x + y \leq 6$$

$$x - y \leq 6$$

$$x \geq 0$$

$$y \geq 0$$

Which point maximizes the objective function
 $P(x,y) = 3x + 4y$?

A. (0,0)

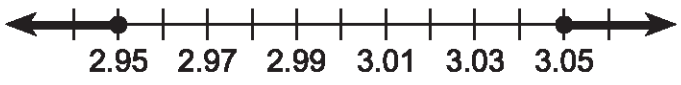
B. (0,6)

C. (4,0)

D. (5,1)

16) Mark works in a factory as a quality-control engineer. The inequality $|w - 3| < 0.05$ represents all acceptable widths, w cm, for a particular part that is made in the factory.

- A. Explain what the inequality means in terms of the acceptable widths of the part.
- B. Mark solves the inequality and graphs the solution set. Explain what Mark did incorrectly.

| | |
|--|--|
| 1. $w + 3 < -0.05$ and $w + 3 > 0.05$ | I removed the absolute value bars and changed $w - 3$ to $w + 3$ since the expression was in absolute value bars. I used -0.05 with the less than sign and 0.05 with the greater than sign. |
| 2. $w < 3 - 0.05$ and $w > 3 + 0.05$ | I added 3 to both sides of each inequality. |
| 3. $w < 2.95$ and $w > 3.05$ | I simplified the right sides of the inequalities. |
| 4.  | I used closed dots since there are less than and greater than signs. I shaded to the outsides because the less than sign tells me to go to the left of 2.95 and the greater than sign tells me to go to the right of 3.05. |

- C. Solve and graph the correct solution set of the inequality. Show your work, and explain why your solution is correct.

Scoring Criteria:

16)

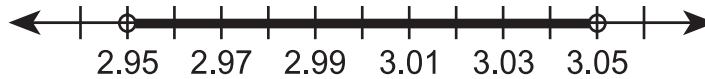
A 4-point response may include, but is not limited to, the following points:

- A. **Correct explanation of the inequality:** All acceptable widths are within 0.05 cm of 3 cm.

- B. **Explanation of what Mark did incorrectly:** In step 1, Mark used $w + 3$ instead of $w - 3$ and 0.05 and -0.05 are in the wrong inequalities. First, Mark should have written the original inequality without the absolute value bars. Then, Mark should have rewritten the original inequality, changing the inequality sign to greater than and the right-hand side of inequality to -0.05 . In step 2, Mark added 3 to the right-hand side of both inequalities but subtracted 3 from the left-hand side of both inequalities. He should have subtracted 3 from both sides of the inequalities. In step 4, Mark used closed dots instead of open dots. Closed dots indicate that 2.95 and 3.05 are part of the solution set. However, these values should not be included in the solution set.

- C. **Correct solution to the inequality:** $2.95 < w < 3.05$

Correct graph:



Appropriate work needed to find the answer:

$$-0.05 < w - 3 < 0.05$$

$$3 - 0.05 < w < 3 + 0.05$$

Explanation of why the solution is correct: I made a compound inequality with -0.05 on the left of the given inequality and then removed the absolute value bars. Then, I added 3 to each of the 3 parts of the inequality. I put open dots at 2.95 and 3.05 because inequality uses less than signs instead of less than or equal to signs. I shaded between 2.95 and 3.05 because there are less than signs instead of greater than signs.

Note: In Part C, the student does not have to explain the things that Mark has already done correctly in Part B.

Rubric:

- 4 A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**
- The response addresses all parts of the question or problem correctly.
 - The response demonstrates efficient and accurate use of appropriate procedures.
 - The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
 - The explanation in the response is clear and coherent.
- 3 A response at this level provides evidence of competent knowledge and understanding of the subject matter.**
- The response addresses most parts of the question or problem correctly.
 - The response includes some minor errors but generally uses appropriate procedures accurately.
 - The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
 - The explanation in the response is mostly clear and coherent.
- 2 A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**
- The response addresses some parts of the question or problem correctly.
 - The response includes a number of errors but demonstrates some use of appropriate procedures.
 - The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
 - The explanation in the response is partially clear, but some parts may be difficult to understand.
- 1 A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**
- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
 - The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
 - The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
 - Many parts of the explanation are difficult to understand.
- 0 A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

QualityCore® Benchmark Assessment

Algebra II – Benchmark 2 Number Sense and Operation Skills; Quadratic Functions

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|-------|-----|-----------------|----------------|
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| 00435 | B | L1 | E.1.c |
| 00375 | D | L1 | E.2.a |
| 00395 | C | L1 | E.2.b |
| 00345 | A | L1 | E.3.a |
| 00268 | A | L1 | E.3.b |
| 00564 | B | L2 | C.1.c |
| 00562 | A | L2 | C.1.b |
| 00567 | D | L2 | C.1.d |
| 00340 | C | L2 | E.1.a |
| 00358 | D | L2 | E.1.b |
| 00290 | C | L2 | E.2.b |
| 00425 | B | L2 | E.2.c |
| 00304 | C | L2 | E.3.b |
| 00361 | A | L2 | E.3.c |
| 00449 | C | L3 | E.1.a |
| 00342 | B | L3 | E.1.d |
| 00537 | B | L3 | E.2.c |
| 00294 | D | L3 | E.3.c |
| 00362 | A | L3 | E.3.d |
| 01005 | - | L3 | C.1.d E.2.a |

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Name:
Teacher:

Date:
Class/Period:

1) What is the complex conjugate of $7 + \sqrt{-8}$?

- A. $7 + 4i\sqrt{2}$
- B. $7 - 4i\sqrt{2}$
- C. $7 + 2i\sqrt{2}$
- D. $7 - 2i\sqrt{2}$

2) What set of conditions will yield complex conjugate zeros of a quadratic function $f(x) = ax^2 + bx + c$?

- A. $-b < b^2$
- B. $b^2 < 4ac$
- C. $2a < 0$
- D. $b^2 > 4ac$

3) Which equation has the same range as $y = (x + 3)^2$?

- A. $y = (x + 3)^2 - 2$
- B. $y = x^2 + 9$
- C. $y = 2(x - 3)^2 + 1$
- D. $y = (x - 5)^2$

4) Which equation is the reflection of $y = x^2 - 4x + 3$ across the x-axis?

- A. $y = x^2 - 4x + 3$
- B. $y = x^2 - 4x - 3$
- C. $y = -x^2 + 4x - 3$
- D. $y = -x^2 + 4x + 3$

5) What are the x -intercepts of the ellipse with equation $9x^2 + 4y^2 = 36$?

- A. 2, -2
- B. 3, -3
- C. 4, -4
- D. 6, 0

6) What is the equation of the circle with center $(3, -6)$ and radius $\frac{15}{8}$?

- A. $(x - 3)^2 + (y + 6)^2 = \frac{225}{64}$
- B. $(x - 3)^2 + (y + 6)^2 = \frac{15}{8}$
- C. $(x + 3)^2 + (y - 6)^2 = \frac{225}{64}$
- D. $(x + 3)^2 + (y - 6)^2 = \frac{15}{8}$

7) Write $\frac{9-i^2}{3-i}$ in standard form.

- A. $3 - i$
- B. $3 + i$
- C. $\frac{10}{3}$
- D. $\frac{8}{3}$

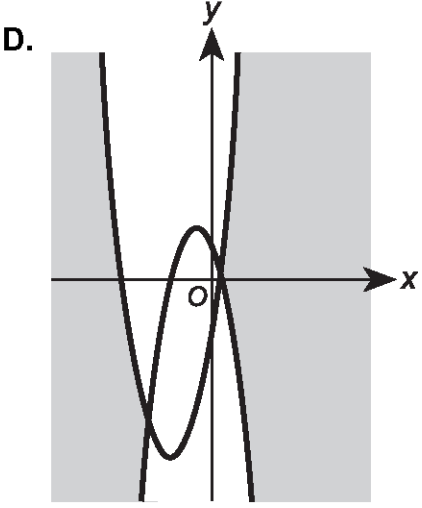
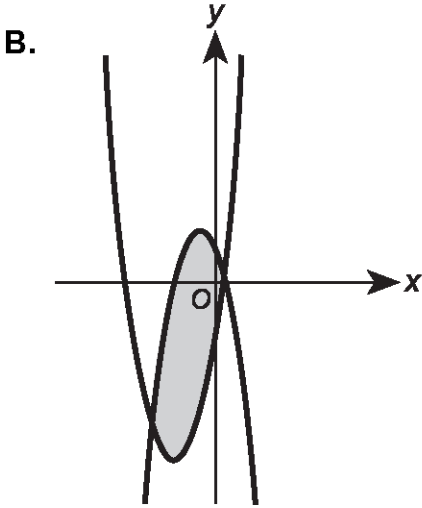
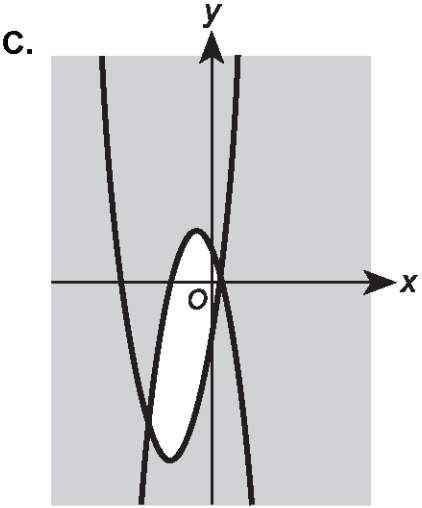
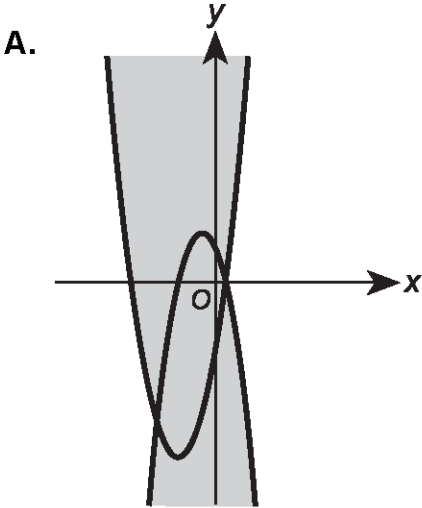
8) If $c - d = 7$ and $c = 3 - 4i$, what is d ?

- A. $-4 - 4i$
- B. $-4 + 4i$
- C. $4 - 4i$
- D. $4 + 4i$

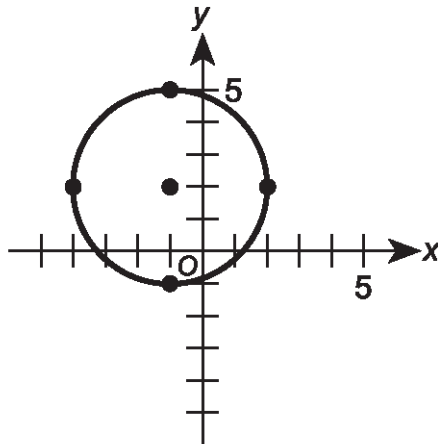
- 9) Given $f(x) = \sqrt{2x-6}$ and $g(x) = \frac{5}{x^2-9}$, what is the domain of $f(g(x))$?
- A. $x \neq 3, x \neq -3$
 - B. $x \neq 9, x \neq -9$
 - C. $x < -3$
 - D. $x > 3$
- 10) Monte has a small herb garden that has a length 3 yd longer than twice its width. If the area of the garden is 50 yd^2 , what is the length of the garden, to the nearest tenth of a yard?
- A. 4.3
 - B. 7.3
 - C. 11.6
 - D. 37.4
- 11) For the equation $x^2 - 4x + 4 = 9$, determine the discriminant.
- A. -36
 - B. 0
 - C. 6
 - D. 36
- 12) Which transformations can be performed on the graph of $f(x) = x^2$ that result in the graph of $f(x) = -2x^2 - 12x - 13$?
- A. Shift left 3 units, stretch horizontally by a factor of 2, reflect through the y-axis, and shift down 5 units
 - B. Shift right 3 units, stretch horizontally by a factor of 2, reflect through the y-axis, and shift down 5 units
 - C. Shift left 3 units, stretch vertically by a factor of 2, reflect through the x-axis, and shift up 5 units
 - D. Shift right 3 units, stretch vertically by a factor of 2, reflect through the x-axis, and shift down 5 units

13) Which graph represents the solution set to this system of inequalities?

$$\begin{cases} y \geq x^2 + 6x - 5 \\ y \leq -x^2 - 2x + 3 \end{cases}$$



14) What is the equation of this circle?



- A. $(x + 1)^2 + (y - 2)^2 = 6$
- B. $(x - 1)^2 + (y + 2)^2 = 6$
- C. $(x + 1)^2 + (y - 2)^2 = 9$
- D. $(x - 1)^2 + (y + 2)^2 = 9$

15) What is the standard form of this equation of a circle?

$$3x^2 + 3y^2 - 6x + 18y + 18 = 0$$

- A. $(x - 1)^2 + (y + 3)^2 = 4$
- B. $(x + 1)^2 + (y - 3)^2 = 4$
- C. $(x - 2)^2 + (y + 6)^2 = 34$
- D. $(x + 2)^2 + (y - 6)^2 = 34$

16) Given $(k - 1)x^2 + kx + 1 = 0$, where -1 is one solution, what is the other solution?

- A. 1
- B. $-\frac{1}{3}$
- C. $\frac{-1}{k-1}$
- D. $\frac{-k(k-2)}{2(k-1)}$

17) Given $x > 0$, at which value of x will $y_2 - y_1 = 2$?

$$y_1 = 2x^2 - 2x + 5$$

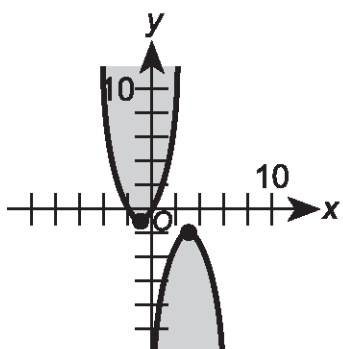
$$y_2 = 3x^2 - 5x + 7$$

- A. 2
- B. 3
- C. 4
- D. 5

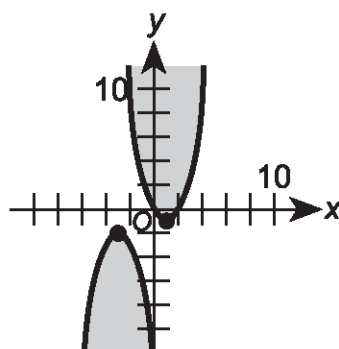
18) Which graph represents the solution set to this system of quadratic inequalities?

$$\begin{cases} y \geq -(x-3)^2 - 2 \\ y \leq (x+1)^2 - 1 \end{cases}$$

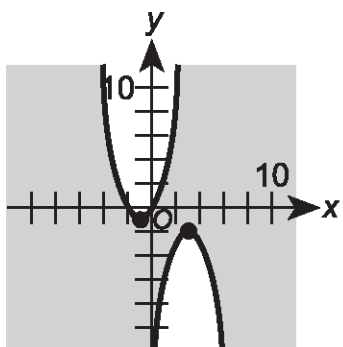
A.



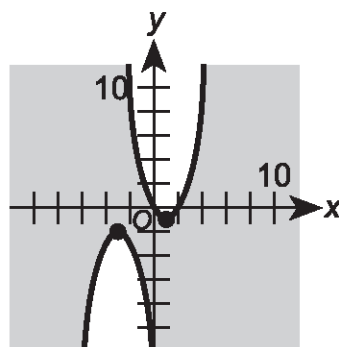
C.



B.



D.



19) The equation of a circle that is tangent to both the x -axis and the y -axis is given as $(x - a)^2 + (y - b)^2 = c$. What must always be true about the values of a , b , and c ?

- A. $a = b$ and $a^2 + b^2 = c$
- B. $a = b$ and $c = a^2$
- C. $|a| = |b|$ and $a^2 + b^2 = c$
- D. $|a| = |b|$ and $c = a^2$

20) Which equation represents a parabola with directrix $x = 6$ and focus $(2, -3)$?

- A. $x = -\frac{1}{8}(y + 3)^2 + 4$
- B. $x = -\frac{1}{16}(y + 3)^2 + 2$
- C. $y = -\frac{1}{8}(x - 2)^2 + 4\frac{1}{2}$
- D. $y = -\frac{1}{6}(x - 2)^2 + 4\frac{1}{2}$

21) Consider the real-valued functions $f(x) = x^2 - 6$ and $g(x) = 2x - 3$.

- A. What are the domain and range of each function? Explain how you found your answers.
- B. Find $f(g(x))$. Show your algebraic work, and explain how you found your answer.
- C. What are the domain and range of $f(g(x))$? Show your algebraic work, and explain how you found your answers.

Scoring Criteria:

21) A 4-point response may include, but is not limited to, the following points:

A. **Correct domain for $f(x)$:** All real numbers

Correct range for $f(x)$: $y \geq -6$

Correct domain for $g(x)$: All real numbers

Correct range for $g(x)$: All real numbers

Explanation of how the answer was found: The domain is all real numbers because any number can be substituted for x in $f(x)$. The range is $y \geq -6$ because $f(x)$ is a parabola with the vertex at $(0, -6)$. Since the coefficient on the x^2 term is positive, the parabola opens upward, and the range will be all numbers greater than or equal to the y -value of the vertex, -6 . The domain is all real numbers because any number can be substituted for x in $g(x)$. The range is all real numbers because $g(x)$ is a linear function and each y -value is used in a linear function.

B. **Correct expression for $f(g(x))$:** $4x^2 - 12x + 3$

Appropriate work leading to the answer:

$$f(g(x)) = f(2x - 3) = (2x - 3)^2 - 6 = 4x^2 - 12x + 9 - 6$$

Explanation of how the answer was found: I substituted in $2x - 3$ for $g(x)$. Then, I substituted $2x - 3$ for each x in $g(x)$. I squared $2x - 3$ using FOIL, then simplified and subtracted 6.

C. **Correct domain for $f(g(x))$:** All real numbers

Correct range for $f(g(x))$: $y \geq -6$

Appropriate work leading to the answer:

$$x = \frac{-(-12)}{2(4)} = \frac{12}{8} = \frac{3}{2}$$

$$f(g(x)) = f\left(g\left(\frac{3}{2}\right)\right) = 4\left(\frac{3}{2}\right)^2 - 12\left(\frac{3}{2}\right) + 3 = 4\left(\frac{9}{4}\right) - 18 + 3 = 9 - 15 = -6$$

Explanation of how the answer was found: The domain is all real numbers because any number can be substituted for x in $f(g(x))$. Also, the domain of $g(x)$, the inner function, is all real numbers. The range is $y \geq -6$ because $f(g(x))$ is a parabola with its vertex at $\left(\frac{3}{2}, -6\right)$. Since the coefficient on the x^2 term is positive, the parabola opens upward, and the range will be all numbers greater than or equal to the y -value of the vertex, -6 .

Rubric:

- 4 A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**
- The response addresses all parts of the question or problem correctly.
 - The response demonstrates efficient and accurate use of appropriate procedures.
 - The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
 - The explanation in the response is clear and coherent.
- 3 A response at this level provides evidence of competent knowledge and understanding of the subject matter.**
- The response addresses most parts of the question or problem correctly.
 - The response includes some minor errors but generally uses appropriate procedures accurately.
 - The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
 - The explanation in the response is mostly clear and coherent.
- 2 A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**
- The response addresses some parts of the question or problem correctly.
 - The response includes a number of errors but demonstrates some use of appropriate procedures.
 - The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
 - The explanation in the response is partially clear, but some parts may be difficult to understand.
- 1 A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**
- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
 - The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
 - The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
 - Many parts of the explanation are difficult to understand.
- 0 A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

QualityCore[®] Benchmark Assessment

Algebra II – Benchmark 3 Polynomial Functions

The following pages contain one of the Benchmark Assessments for this course. The table below gives the ID number for each item, the correct answer (Key), the cognitive level, and the alphanumeric code for each ACT Course Standard measured by the item. (The language associated with each code appears in the *ACT Course Standards* document for this course.)

The items in this PDF file appear in the order presented in the table. Multiple-choice (MC) directions follow the table and are followed by a name sheet and the MC items.

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| ID | Key | Cognitive Level | Standard |
|-------|-----|-----------------|----------|
| 00401 | A | L1 | F.1.a |
| 00428 | C | L1 | F.1.a |
| 00441 | D | L1 | F.2.b |
| 00386 | D | L2 | F.1.a |
| 00387 | B | L2 | F.1.b |
| 00363 | D | L2 | F.1.b |
| 00322 | C | L2 | F.2.a |
| 00456 | B | L2 | F.2.b |
| 00458 | A | L2 | F.2.c |
| 00376 | D | L2 | F.2.c |
| 00364 | C | L3 | F.1.b |
| 00574 | A | L3 | F.2.a |
| 00538 | A | L3 | F.2.b |
| 00297 | C | L3 | F.2.c |
| 00459 | B | L3 | F.2.d |
| 01010 | - | L3 | F.1.b |

Directions:

Solve each problem, choose the best answer, and then circle the corresponding letter. Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.

You are permitted to use a calculator on this test. You may use your calculator for any problems you choose, but some of the problems may best be solved without using a calculator.

Note: Unless otherwise stated, all of the following assumptions apply to these problems.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word *line* indicates a straight line.
4. The word *average* indicates the arithmetic mean.

Name:
Teacher:

Date:
Class/Period:

1) Evaluate this function for $x = -3$.

$$f(x) = 4x^3 - 5x^2 + 2x - 4$$

- A. -163
- B. -151
- C. -73
- D. 53

2) Simplify this expression:

$$3(x^2 + 2) - 5(2x^2 + 3x - 4) + 2(-x^2 - 4)$$

- A. $-5x^2 - 15x + 18$
- B. $-5x^2 + 15x - 22$
- C. $-9x^2 - 15x + 18$
- D. $-9x^2 + 15x - 22$

3) Find all zeros of $f(x) = x^4 - 1$.

- A. $x = -1, 1$
- B. $x = 1, i$
- C. $x = 1, i, -i$
- D. $x = 1, -1, i, -i$

4) What is the completely simplified expression of

$$\frac{x^3 - 4x}{x^5 - 8x^2} ?$$

- A. $\frac{x^3 - 4x}{x^5 - 8x^2}$
- B. $\frac{1}{x(x+2)}$
- C. $\frac{x^2 - 4}{x(x^3 - 8)}$
- D. $\frac{x+2}{x(x^2 + 2x + 4)}$

5) The area of a right triangle is $\frac{1}{2}x^2 - 5x + 12$. If one leg is $x - 4$, what is the other leg?

- A. $x - 3$
- B. $x - 6$
- C. $\frac{1}{2}x - 3$
- D. $\frac{1}{2}x - 4 + \frac{4}{x-2}$

6) Which expression is the completely factored form of $2x^5 - 58x^3 + 200x$?

- A. $2x(x^4 - 29x^2 + 100)$
- B. $2x(x^2 - 4)(x^2 - 25)$
- C. $2(x + 2)(x - 2)(x + 5)(x - 5)$
- D. $2x(x + 2)(x - 2)(x + 5)(x - 5)$

7) How many real zeros does $h(t)$ have?

$$h(t) = 4t^3 - 2t^2 + t - 10$$

- A. 3
- B. 2
- C. 1
- D. 0

8) What are the possible rational solutions of this equation?

$$\frac{1}{2}x^5 + 3x^4 - 4x^3 + 6x = 2$$

- A. $\pm 1, \pm 2$
- B. $\pm 1, \pm 2, \pm 4$
- C. $\pm 1, \pm 2, \pm 3, \pm 6$
- D. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

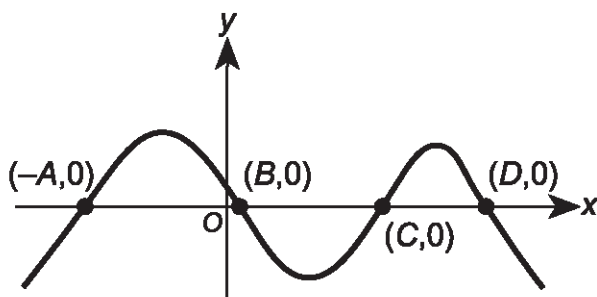
- 9) A certain seventh degree polynomial function can be factored as:

$$(x + 2)^2(x^2 + 1)(x - 5)^3$$

How many x-intercepts does this function have?

- A. 2
- B. 3
- C. 5
- D. 7

- 10) Which equation represents this graph?



- A. $y = (x + D)^2(x + C)^2(x + B)^2(x - A)^2$
- B. $y = (x + D)(x + C)(x + B)(x - A)$
- C. $y = (x - D)^2(x - C)^2(x - B)^2(x + A)^2$
- D. $y = (x - D)(x - C)(x - B)(x + A)$

- 11) Completely factor this expression.

$$128x^7y + 32x^4y^4 + 2xy^7$$

- A. $2xy(8x^3 + y^3)^2$
- B. $2xy(64x^6 + 16x^3y^3 + y^6)$
- C. $2xy(2x + y)^2(4x^2 - 2xy + y^2)^2$
- D. $2xy(2x + y)^2(4x^2 + 2xy + y^2)^2$

- 12) How many rational zeros does this function have?

$$f(x) = (x^2 + 4)(x - 4)(x^2 - 2x - 5)$$

- A. 1
- B. 2
- C. 3
- D. 5

13) What are the rational roots of the function

$$f(x) = x^4 + 2x^2 - 3 ?$$

- A. -1, 1
- B. -1, 1, 3, -3
- C. -1, 1, $\sqrt{3}$, $-\sqrt{3}$
- D. 3, -3

14) Which third degree polynomial has 3 and $3 - i$ as zeros?

- A. $x^3 - 3x^2 - 9x + 27$
- B. $x^3 + 3x^2 - 10x - 30$
- C. $x^3 - 9x^2 + 28x - 30$
- D. $x^3 + 9x^2 + 28x + 30$

15) This polynomial equation

$$3x^3 - 8x^2 - 185x + 126 = 0$$

has 3 real solutions. Two of the solutions are integers but the other is not. The noninteger solution is between which 2 consecutive integers?

- A. -1 and 0
- B. 0 and 1
- C. 5 and 6
- D. 8 and 9

16) Use polynomial long division to find $\frac{3x^4 - x^5 + 4x^2 - 7 + 2x}{x^2 - 5x + 2}$. Show your work algebraically. Explain how you found your answer as if you were writing to a student who was not in class when the concept was taught.

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QualityCore® Benchmark Assessment

Algebra II – Benchmark 4 Nonpolynomial Functions

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|-------|-----|-----------------|----------------|
| 00310 | B | L1 | G.2.b |
| 00368 | A | L1 | G.3.a |
| 00271 | A | L1 | G.3.b |
| 00328 | D | L1 | G.3.e |
| 00313 | A | L1 | G.3.d |
| 00274 | B | L1 | G.3.f |
| 00388 | C | L2 | G.1.a |
| 00405 | C | L2 | G.1.b |
| 00348 | B | L2 | G.1.d |
| 00406 | B | L2 | G.1.e |
| 00367 | C | L2 | G.1.g |
| 00311 | B | L2 | G.3.b |
| 00429 | A | L2 | G.3.d |
| 00312 | B | L2 | G.3.c |
| 00540 | D | L3 | G.1.a |
| 00541 | B | L3 | G.1.d |
| 00577 | A | L3 | G.1.g |
| 00309 | C | L3 | G.2.a |
| 00430 | D | L3 | G.3.a |
| 00314 | C | L3 | G.3.g |
| 01017 | - | L3 | G.3.e G.3.f |

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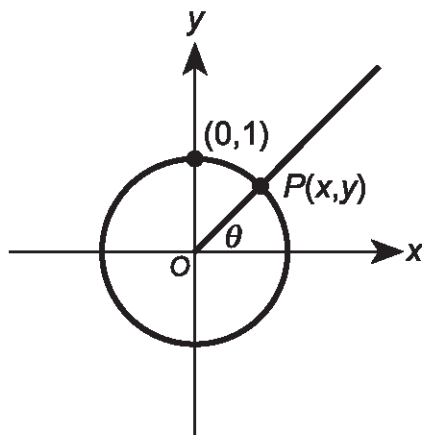
1) If $\log_2 x = -3$, what is x ?

- A. $\frac{1}{9}$
- B. $\frac{1}{8}$
- C. 8
- D. 9

2) In $\triangle ABC$, $m\angle ACB = 48^\circ$, $AC = 17$ ft, and $CB = 10$ ft.
To the nearest tenth of a foot, what is AB ?

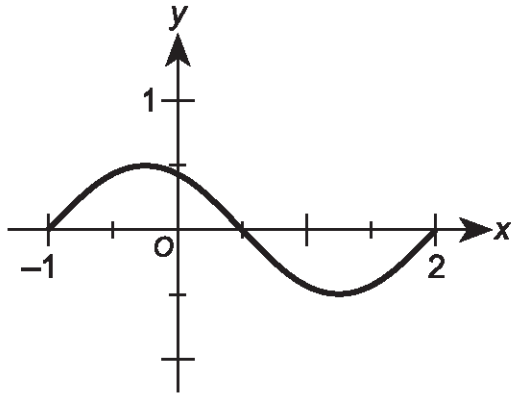
- A. 12.7
- B. 13.7
- C. 19.7
- D. 25.1

3) In this unit circle where θ is the angle formed by the x -axis and \vec{OP} , what is another way to write the coordinates of P ?



- A. $(\cos \theta, \sin \theta)$
- B. $(\sin \theta, \cos \theta)$
- C. $(\cot \theta, \tan \theta)$
- D. $(\tan \theta, \cot \theta)$

4) Which best describes the range of this graph?



- A. $-1 \leq x \leq 2$
- B. $-\frac{1}{2} \leq x \leq \frac{1}{2}$
- C. $-1 \leq y \leq 2$
- D. $-\frac{1}{2} \leq y \leq \frac{1}{2}$

5) Where do the minimum values of $y = \sin x$ occur on $[-2\pi, 2\pi]$?

- A. $-\frac{\pi}{2}$ and $\frac{3\pi}{2}$
- B. $-\frac{3\pi}{2}$ and $\frac{\pi}{2}$
- C. $-\pi$ and π
- D. -2π and 2π

6) What are the amplitude and period of $y = 2 \cos x$?

- A. Amplitude is -2 ; period is 2
- B. Amplitude is 2 ; period is 2π
- C. Amplitude is 2 ; period is 4π
- D. Amplitude is 2 ; period is 2

- 7) One printer takes 3 hours to complete a job. Another printer can do the same job in 4 hours. When the job runs on both printers, how many hours will it take to complete?

- A. $\frac{7}{24}$
- B. $\frac{7}{12}$
- C. $\frac{12}{7}$
- D. $\frac{24}{7}$

- 8) Given $x > 0$, $y > 0$, which expression is equivalent to:

$$\frac{\sqrt[3]{27x^5}\sqrt[4]{32y^4}}{\sqrt{8x^8y^6}}$$

- A. $\frac{3\sqrt[3]{x^2}\sqrt[4]{4}}{2x^3y^2}$
- B. $\frac{3\sqrt{2}\sqrt[3]{x^2}\sqrt[4]{2}}{2x^3y^2}$
- C. $\frac{3\sqrt[3]{x^2}\sqrt[4]{2}}{\sqrt{2}x^3y^2}$
- D. $\frac{18\sqrt[3]{x^2}}{x^3y^2}$

- 9) Rationalize the denominator and simplify: $\frac{4+\sqrt{6}}{4+\sqrt{2}}$

- A. $\frac{8-\sqrt{2}}{7}$
- B. $\frac{8-2\sqrt{2}-\sqrt{3}+2\sqrt{6}}{7}$
- C. $8+2\sqrt{2}-\sqrt{3}+2\sqrt{6}$
- D. $\frac{16-4\sqrt{2}-2\sqrt{3}+4\sqrt{6}}{14}$

- 10) Rewrite this expression in simplest form. Assume variables represent positive numbers.

$$\sqrt{\frac{16w^2y^3}{75x^5}}$$

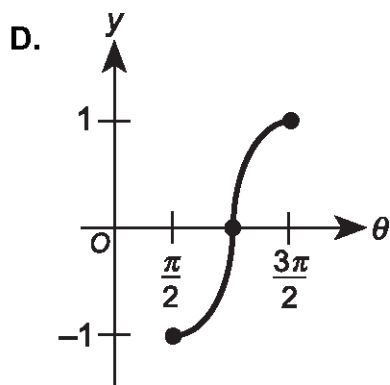
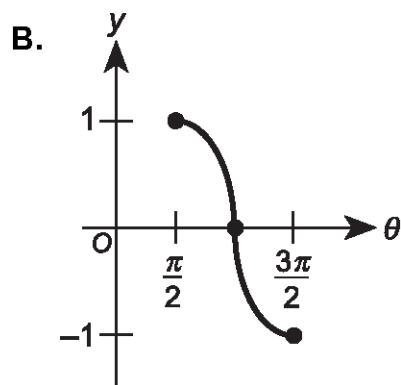
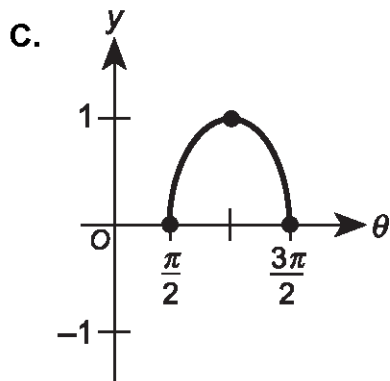
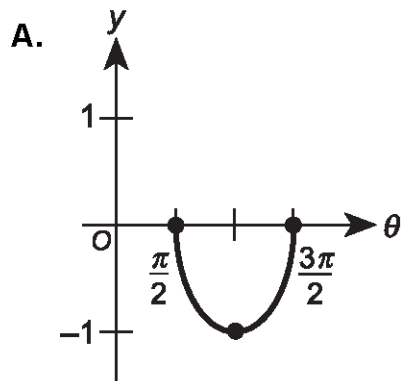
- A. $\frac{\sqrt{3xy}}{3x}$
- B. $\frac{4wy\sqrt{3xy}}{15x^3}$
- C. $\frac{4wy^2\sqrt{5xy}}{25x^3}$
- D. $\frac{4wy^2\sqrt{3xy}}{15x^3}$

- 11) Mr. Johnson bought a conical camping tent for his 2 daughters. The radius of the circular base of the tent measures 4.5 ft, and the tent's lateral surface area is 110.6 ft². Use the formula $S = \pi r\sqrt{r^2 + h^2}$, where S is the lateral surface area and r is the radius, to find the height, h , of the tent, to the nearest tenth of a foot.

(Note: $\pi \approx 3.14$)

- A. 3.3
- B. 6.3
- C. 6.4
- D. 8.2
- 12) An angle, A , in standard position on the unit circle, has its terminal side in Quadrant III. If $\tan A = \frac{4}{3}$, what is $\sin A$?
- A. $-\frac{3}{5}$
- B. $-\frac{4}{5}$
- C. $\frac{3}{5}$
- D. $\frac{4}{5}$

13) Graph $y = \cos \theta$ when $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$.



14) Which is the radian equivalent to three and one-quarter revolutions clockwise?

A. $-\frac{13\pi}{4}$

B. $-\frac{13\pi}{2}$

C. $\frac{13\pi}{4}$

D. $\frac{13\pi}{2}$

15) A business owner spent \$500 on start-up fees to produce and sell candles. Each candle costs an additional \$3.00 to produce. What is the minimum number of candles that the owner must produce for the average cost per candle to be less than \$3.75 ?

- A. 134
- B. 167
- C. 375
- D. 667

16) Variables a , b , and c are real numbers where $b = c^2$ and $a = b^3$. Write $\sqrt[3]{ac} + \sqrt[3]{b^2}$ in terms of c .

- A. $(2c^2)\sqrt[3]{c}$
- B. $c^2\sqrt[3]{c} + c\sqrt[3]{c}$
- C. $2c\sqrt[3]{c}$
- D. $(c^6 + c)\sqrt[3]{c}$

17) Given:

$$b = \sqrt{a}$$

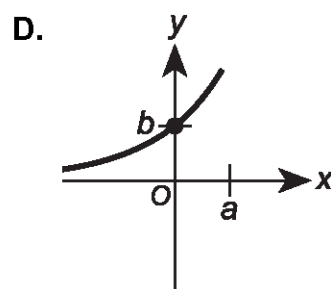
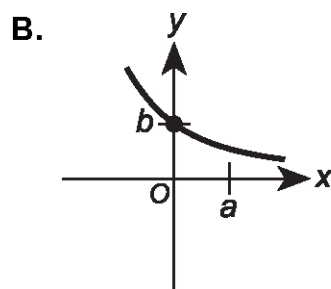
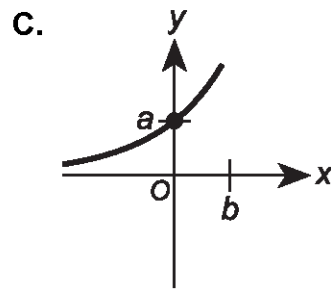
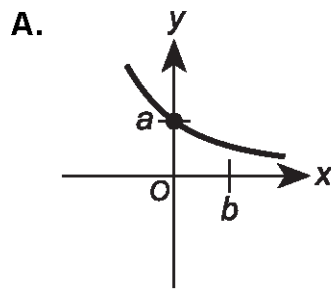
$$c = a^2$$

$$d = b^2c$$

Which expression is equivalent to $\frac{d^2b}{ac}$ in terms of a ?

- A. $a^3\sqrt{a}$
- B. $a^7\sqrt{a}$
- C. a^9
- D. a^{12}

18) Which is the graph of $y = ab^x$ where $a > 0$ and $b > 1$?



19) Jennifer and Robbie stand 50 ft apart on opposite sides of a statue. The angle of elevation from Jennifer's feet to the top of the statue is 46° , while the angle of elevation from Robbie's feet to the top of the statue is 52° . How tall, to the nearest tenth of a foot, is the statue?

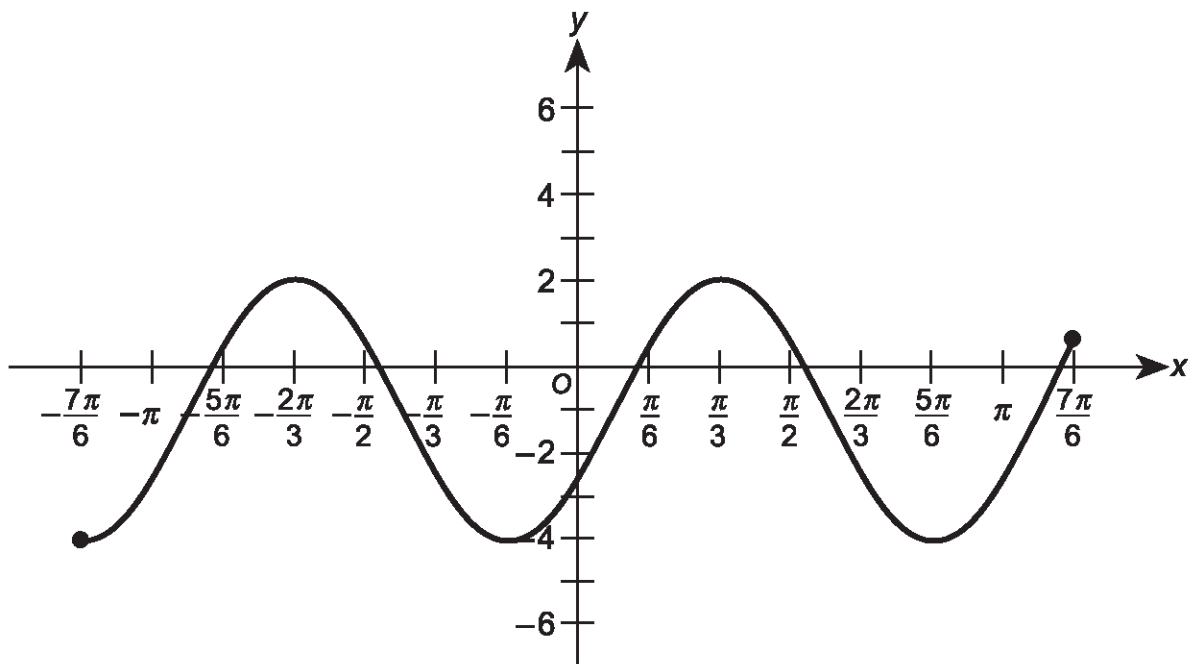
(Note: Assume the statue and both people stand on level ground.)

- A. 22.4
- B. 25.9
- C. 26.4
- D. 28.6

20) What is the equation for the asymptote of $y = \tan(\theta - \frac{\pi}{4}) + \frac{\pi}{2}$ on the interval $[0, \pi]$?

- A. $x = \frac{\pi}{4}$
- B. $x = \frac{\pi}{2}$
- C. $x = \frac{3\pi}{4}$
- D. $x = \pi$

21) Consider the graph of the periodic function shown:



- What is the period of the function? Show your work, and explain how you found your answer.
- What is the amplitude of the graph of the function? Show your work, and explain how you found your answer.
- What is the domain of the graphed function? Explain how you found your answer.
- What is the range of the graphed function? Explain how you found your answer.

Scoring Criteria:

21)

A 4-point response may include, but is not limited to, the following points:

A. Correct period: π

Appropriate work needed to find the answer:

$$\frac{\pi}{3} - \left(-\frac{2\pi}{3}\right) = \frac{3\pi}{3}$$

Explanation of how the answer was found: The graph has successive maxima at $x = -\frac{2\pi}{3}$ and $x = \frac{\pi}{3}$. Therefore, the graph goes through 1 period in going from $x = -\frac{2\pi}{3}$ to $x = \frac{\pi}{3}$.

B. Correct amplitude: 3

Appropriate work needed to find the answer:

$$\frac{1}{2}(2 - (-4)) = \frac{1}{2}(6)$$

Explanation of how the answer was found: The amplitude is $\frac{1}{2}$ the distance from the maximum (2) to the minimum (-4).

C. Correct domain: $\left[-\frac{7\pi}{6}, \frac{7\pi}{6}\right]$

Explanation of how the answer was found: The graph starts at $x = -\frac{7\pi}{6}$ and ends at $x = \frac{7\pi}{6}$. Since there are closed dots at these points, I include the endpoints of the domain.

D. Correct range: $[-4, 2]$

Explanation of how the answer was found: The minimum y-value on the graph is 4, and the maximum value on the graph is 2. Since there are closed dots at these points, I include the endpoints of the range.

Rubric:

- 4 A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**
- The response addresses all parts of the question or problem correctly.
 - The response demonstrates efficient and accurate use of appropriate procedures.
 - The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
 - The explanation in the response is clear and coherent.
- 3 A response at this level provides evidence of competent knowledge and understanding of the subject matter.**
- The response addresses most parts of the question or problem correctly.
 - The response includes some minor errors but generally uses appropriate procedures accurately.
 - The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
 - The explanation in the response is mostly clear and coherent.
- 2 A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**
- The response addresses some parts of the question or problem correctly.
 - The response includes a number of errors but demonstrates some use of appropriate procedures.
 - The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
 - The explanation in the response is partially clear, but some parts may be difficult to understand.
- 1 A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**
- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
 - The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
 - The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
 - Many parts of the explanation are difficult to understand.
- 0 A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

QualityCore[®] Benchmark Assessment

Algebra II – Benchmark 5 Probability; Sequences and Series

The following pages contain one of the Benchmark Assessments for this course. The table below gives the ID number for each item, the correct answer (Key), the cognitive level, and the alphanumeric code for each ACT Course Standard measured by the item. (The language associated with each code appears in the *ACT Course Standards* document for this course.)

The items in this PDF file appear in the order presented in the table. Multiple-choice (MC) directions follow the table and are followed by a name sheet and the MC items.

Following the MC items, you will find a constructed-response (CR) item followed by its scoring criteria and/or scoring rubric. **DO NOT DISTRIBUTE SCORING CRITERIA TO STUDENTS.** The scoring rubric can be included or excluded at your discretion.

| ID | Key | Cognitive Level | Standard |
|-------|-----|-----------------|----------------|
| 00275 | D | L1 | H.1.a |
| 00420 | C | L1 | H.1.c |
| 00330 | C | L1 | H.1.f |
| 00417 | A | L1 | H.2.a |
| 00351 | D | L2 | H.1.b |
| 00421 | B | L2 | H.1.d |
| 00329 | B | L2 | H.1.e |
| 00277 | C | L2 | H.2.e |
| 00331 | B | L2 | H.2.a |
| 00333 | B | L2 | H.2.c |
| 00409 | A | L3 | H.1.a |
| 00316 | D | L3 | H.1.b |
| 00332 | A | L3 | H.2.b |
| 00319 | C | L3 | H.2.c |
| 00370 | A | L3 | H.2.d |
| 01018 | - | L3 | H.1.a H.1.b |

Directions:

Solve each problem, choose the best answer, and then circle the corresponding letter. Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.

You are permitted to use a calculator on this test. You may use your calculator for any problems you choose, but some of the problems may best be solved without using a calculator.

Note: Unless otherwise stated, all of the following assumptions apply to these problems.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word *line* indicates a straight line.
4. The word *average* indicates the arithmetic mean.

Name:
Teacher:

Date:
Class/Period:

1) Richard has 5 shirts, 6 pairs of jeans, and 3 vests. How many different outfits, each composed of a shirt, a pair of jeans, and a vest, can he make?

- A. 6
- B. 14
- C. 33
- D. 90

2) A high school's enrollment is 27% juniors and 31% seniors. What is the probability that a student chosen at random from this high school will be a junior or a senior?

- A. $\frac{1}{25}$
- B. $\frac{2}{25}$
- C. $\frac{29}{50}$
- D. $\frac{87}{100}$

3) Yi has a bag of 15 stones: 8 blue and 7 purple. If Yi picks 2 blue stones and does not return them to the bag, what is the probability that she will pick a purple stone next?

- A. $\frac{7}{15}$
- B. $\frac{6}{15}$
- C. $\frac{7}{13}$
- D. $\frac{6}{13}$

4) Find the 100th term of this arithmetic sequence.

5, 8, 11, 14, 17, ...

- A. 302
- B. 305
- C. 308
- D. 311

5) Ten students will participate in a spelling contest. How many outcomes for first, second, and third place are possible?

- A. 30
- B. 90
- C. 120
- D. 720

6) Box A contains marbles: 12 red, 16 blue, 11 green, and 5 yellow. Box B contains chips: 8 red, 7 green, 11 blue, and 1 yellow. If you randomly pick one item from each box, what is the probability that both items will be blue?

- A. $\frac{1}{10}$
- B. $\frac{4}{27}$
- C. $\frac{19}{50}$
- D. $\frac{77}{100}$

7) A standard deck of playing cards has 52 cards. The deck has the same number of black and red cards and has 4 Jacks, two red and two black. What is the probability of randomly picking a Jack or a red card from a standard deck?

- A. $\frac{30}{52}$
- B. $\frac{28}{52}$
- C. $\frac{4}{52}$
- D. $\frac{2}{52}$

- 8) Evaluate $\sum_{x=1}^{10} (7 - 2x)$.
- A. -8
 - B. -13
 - C. -40
 - D. -80
- 9) In a geometric sequence where $a_1 = 15$ and $a_5 = 240$, what is the first term in the sequence that is a multiple of 4 ?
- A. $a_2 = 20$
 - B. $a_3 = 60$
 - C. $a_4 = 60$
 - D. $a_4 = 120$
- 10) On the first of every month, a new library receives a new shipment of 575 book titles. If the library starts the beginning of the first year with 3,000 book titles and does receive a shipment that month, how many book titles will it have at the end of 3 years?
- A. 42,900
 - B. 23,700
 - C. 5,363
 - D. 4,150
- 11) A company with 2000 employees requires that computer passwords consist of 3 digits followed by 3 letters. Each password cannot repeat a letter or number, and no password can be repeated. Passwords must be changed, by the employees, at the beginning of each month. To the nearest year, how many years of passwords exist?
- A. 468
 - B. 732
 - C. 5,616
 - D. 8,788

- 12) This chart shows the number of students, by gender, in each grade at a local high school. The principal will randomly select one student to meet the governor.

| | 9th | 10th | 11th | 12th |
|--------|-----|------|------|------|
| Female | 80 | 95 | 75 | 80 |
| Male | 75 | 100 | 75 | 70 |

Let $A = \{\text{choosing a female}\}$ and $B = \{\text{choosing a ninth grader}\}$. What is $P(B|A)$?

- A. $\frac{8}{65}$
- B. $\frac{31}{66}$
- C. $\frac{16}{31}$
- D. $\frac{8}{33}$
- 13) Iman starts with \$1 on Day 1 and then doubles her money every day thereafter. On which day will she first have more than \$10,000 ?
- A. Day 15
- B. Day 14
- C. Day 10
- D. Day 6
- 14) This table shows some terms of an arithmetic sequence. The sum of the first n terms is 2,464. What is the n th term?

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|---|----|---|---|----|---|----|
| a_n | | 19 | | | 31 | | 39 |

- A. 601
- B. 143
- C. 139
- D. 62

15) Hsiang plans to save all of his earnings for 4 years so that he can buy a used car after he graduates from high school. He receives \$5 for the first week, and earns a \$0.10 increase per week for 2 years, and then a \$0.15 increase per week for the last 2 years. What is the total amount Hsiang will have after 4 years ?

(Note: There are 52 weeks in a year.)

- A. \$3,465.80
- B. \$3,211.00
- C. \$2,402.40
- D. \$1,315.50

16) Javier has 10 marbles in a bag: 5 blue labeled B_1 , B_2 , B_3 , B_4 , and B_5 ; 3 green labeled G_1 , G_2 , and G_3 ; and 2 red labeled R_1 and R_2 . Javier chooses 2 marbles from the bag randomly, one at a time, without putting the chosen marbles back in the bag.

- A. How many combinations exist for these 2 marbles given that the order they are chosen does not matter? Show your work using the Fundamental Counting Principle, and explain why you used the numbers that you did.
- B. List all of the combinations for these 2 marbles in an organized list using the labels listed in the stimulus. Explain how you know you have all the possible combinations.

Scoring Criteria:

16)

A 4-point response may include, but is not limited to, the following points:

A. Correct number of combinations: 45

Appropriate work needed to find the answer:

$$\frac{10(9)}{2}$$

Explanation of how the answer was found: I multiplied the number of marbles by one less than the number of marbles because once I choose one of the 10 marbles, there will only be 9 marbles left from which to choose for the second marble. I then divided this number by 2 since the order in which the marbles is chosen does not matter.

Note: An examinee could also give an explanation of using the formula for combinations.

B. Correct list:

$B_1B_2, B_1B_3, B_1B_4, B_1B_5, B_2B_3, B_2B_4, B_2B_5, B_3B_4, B_3B_5, B_4B_5$

$B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, B_3G_1, B_3G_2, B_3G_3, B_4G_1, B_4G_2, B_4G_3, B_5G_1, B_5G_2, B_5G_3$

$B_1R_1, B_1R_2, B_2R_1, B_2R_2, B_3R_1, B_3R_2, B_4R_1, B_4R_2, B_5R_1, B_5R_2$

G_1G_2, G_1G_3, G_2G_3

$G_1R_1, G_1R_2, G_2R_1, G_2R_2, G_3R_1, G_3R_2$

R_1R_2

Explanation of how the answer was found: I started with the first blue marble and paired it with each other blue marble. Then, I went through the same process starting with the 4 remaining blue marbles. I got 10 combinations. Next, I paired each blue marble with each green marble. I got 15 combinations. Then, I paired each blue marble with each red marble. I got 10 combinations. Next, I paired the 3 green marbles among themselves. I got 3 combinations. Then, I paired each green marble with each red marble. I got 6 combinations. Finally, I paired the red marbles together. I got 1 combination.

Rubric:

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