

Instructional Supports and Resources

K-PREP Sampler Support
Grade 6
Mathematics
8/20/2012

This document provides teachers with instructional supports for effectively teaching the standards that are measured by the sample released K-PREP mathematics items.

Domain:	Statistics and Probability
Cluster	<i>Summarize and describe distributions.</i>
Standards:	6.SP.5c Summarize numerical data sets in relation to their context, such as giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
Standards for Mathematical Practice:	MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure.

Instructional Strategies

This cluster builds on the understandings developed in the Grade 6 cluster “Develop understanding of statistical variability.” Students have analyzed data displayed in various ways to see how data can be described in terms of variability.

Further interpretation of the variability comes from the range and center-of-measure numbers. Prior to learning the computation procedures for finding mean and median, students will benefit from concrete experiences.

To find the median visually and kinesthetically, students should reorder the data in ascending or descending order, then place a finger on each end of the data and continue to move toward the center by the same increments until the fingers touch. This number is the median.

The concept of mean (concept of fair shares) can be demonstrated visually and kinesthetically by using stacks of linking cubes. The blocks are redistributed among the towers so that all towers have the same number of blocks. Students should not only determine the range and centers of measure, but also use these numbers to describe the variation of the data collected from the statistical question asked. The data should be described in terms of its shape, center, spread (range) and interquartile range or mean absolute deviation (the absolute value of each data point from the mean of the data set). Providing activities that require students to sketch a representation based upon given measures of center and spread and a context will help create connections between the measures and real-life situations.

Continue to have students connect contextual situations to data to describe the data set in words prior to computation. Therefore, determining the measures of spread and measures of center mathematically need to follow the development of the conceptual understanding. Students should experience data which reveals both different and identical values for each of the measures. Students need opportunities to explore how changing a part of the data may change the measures of center and measure of spread. Also, by discussing their findings, students will solidify understanding of the meanings of the measures of center and measures of variability, what each of the measures do and do not tell about a set of data, all leading to a better understanding of their usage.

It is important to use the interquartile range in box plots when describing the variation of the data. The mean absolute deviation describes the distance each point is from the mean of that data set. Patterns in the graphical displays should be observed, as should any outliers in the data set. Students should identify the attributes of the data and know the appropriate use of the attributes when describing the data.

Students often use words to help them recall how to determine the measures of center. However, student’s lack of understanding of what the measures of center actually represent tends to confuse them. Median is the number in the middle, but that middle number can only be determined after the data entries are arranged in ascending or descending order. Mode is remembered as the “most,” and often students think this means the largest value, not the “most frequent” entry in the set. Vocabulary is important in mathematics, but conceptual

understanding is equally as important. Usually the mean, mode, or median have different values, but sometimes those values are the same.

Included in the standards document are critical areas for each grade. **Grade 6 CRITICAL AREA OF FOCUS #4:** Students develop understanding of statistical thinking. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Instructional Resources/Tools

[Hollywood Box Office](#) This problem focuses on measures of center and graphical displays.

From the National Council of Teachers of Mathematics, Illuminations: [Height of Students in our Class](#). This lesson has students creating box-and-whisker plots with an extension of finding measures of center and creating a stem-and-leaf plot. This resource continues to support the previous standard, 6.SP.4, displaying numerical data in plots.

<http://illuminations.nctm.org/LessonDetail.aspx?ID=L452>. In this lesson, from Illuminations, students use an online interactive tool to compare and contrast properties of measures of central tendency, specifically the influence of changes in data values on the mean and median. As students change the data values, the interactive figure immediately displays the mean and median of the new data set. Experimenting with this software helps students compare the utility of the mean and the median as measures of center for different data sets.

<http://www.regentsprep.org/regents/math/algebra/AD2/measure.htm> This resource gives definitions and examples for some vocabulary related to this standard.

http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.STAT&ID2=AB.MATH.JR.STAT.CENT&lesson=html/video_interactives/centralTendency/centralTendencyInteractive.html Interactive resource for students to determine what happens to the measures of center when new data is introduced into a data set.

<http://www.youtube.com/watch?v=z9AJk7TvdvQ> This video reviews how to find Mean Absolute Deviation for a set of data

Resources:

[Ohio Department of Education](#). Model Curriculum. March, 2011.

<http://www.education.ohio.gov>

[Arizona Department of Education](#). Mathematics Resources and Common Core Standards. June, 2011.

<http://www.azed.gov/standards-practices/mathematics-standards/>

[North Carolina State Board of Education](#). Elementary and Middle Grades Resources.

<http://www.ncpublicschools.org/curriculum/mathematics/scos/>

[Tools for the Common Core Standards](#). CCSSM Progressions. April, 2011.

<http://commoncoretools.me/category/progressions/>

Domain:	Ratio and Proportional Relationships
Cluster	<i>Understand ratio concepts and use ratio reasoning to solve problems</i>
Standards:	6.RP.3b Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i>
Standards for Mathematical Practice:	MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.4. Model with mathematics MP.5. Use appropriate tools strategically. MP.7. Look for and make use of structure.

Instructional Strategies

Proportional reasoning is a process that requires instruction and practice. It does not develop over time on its own. Grade 6 is the first of several years in which students develop this multiplicative thinking. Examples with ratio and proportion must involve measurements, prices and geometric contexts, as well as rates of miles per hour or portions per person within contexts that are relevant to sixth graders. Experience with proportional and nonproportional relationships, comparing and predicting ratios, and relating unit rates to previously learned unit fractions will facilitate the development of proportional reasoning. Although algorithms provide efficient means for finding solutions, the cross-product algorithm commonly used for solving proportions will not aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage thinking about multiplicative situations instead of indiscriminately applying rules.

Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon and students per bus. For example, 3 cans of pudding cost \$2.48 at Store A and 6 cans of the same pudding costs \$4.50 at Store B. Which store has the better buy on these cans of pudding? Various strategies could be used to solve this problem:

- A student can determine the unit cost of 1 can of pudding at each store and compare.
- A student can determine the cost of 6 cans of pudding at Store A by doubling \$2.48.
- A student can determine the cost of 3 cans of pudding at Store B by taking $\frac{1}{2}$ of \$4.50.

Included in the standards document are critical areas for each grade. **Grade 6 CRITICAL AREA OF FOCUS #1:** Students connect ratio and rate to whole number multiplication and division and use concepts of ratio and rate to solve problems. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

Instructional Resources/Tools

<http://www.mathalicious.com/lesson/big-foot-conspiracy/> In this lesson graphs, unit rates (\$/ounce), and proportions are used to analyze what shoes really cost and to develop an alternate pricing model by which Nike could charge by the ounce. The coordinate graph component could be introduced to give students a glimpse of the progression this cluster will take into 7th grade.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L284>. In this lesson, real-world models are used to develop an understanding of fractions, decimals, unit rates, proportions, and problem solving. The activities in this investigation center on situations involving rational numbers and proportions that students encounter during a fictitious trip to a bakery. These activities involve several important concepts of rational numbers and proportions, including partitioning a unit into equal parts, the quotient interpretation of fractions, the area model

Grade 6 Sampler Item 7

of fractions, determining fractional parts of a unit not cut into equal-sized pieces, equivalence, unit prices, and multiplication of fractions.

<http://illuminations.nctm.org/LessonDetail.aspx?ID=L511> In this lesson, one of a multi-part unit from Illuminations, students calculate unit rates and set up proportions. The students gather data to write and solve their own proportions.

<http://learnzillion.com/lessons/314-solve-rate-and-ratio-word-problems> - In this video, sample situations are given and the problems are solved by simplifying a ratio into a per unit rate.

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Ohio Department of Education. Model Curriculum. March, 2011.
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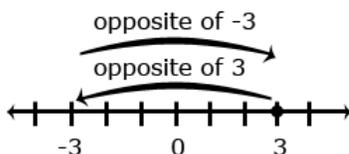
North Carolina State Board of Education. Elementary and Middle Grades Resources.
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Domain:	The Number System
Cluster	<i>Apply and extend previous understandings of numbers to the system of rational numbers.</i>
Standards:	6.NS.6b Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
Standards for Mathematical Practice:	MP.2. Reason abstractly and quantitatively. MP.4. Model with mathematics.

Instructional Strategies

Number lines can be used to show numbers and their opposites. Both 3 and -3 are 3 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids.



Example: Graph the following points in the correct quadrant of the coordinate plane. If you reflected each point across the x-axis, what are the coordinates of the reflected points? What similarities do you notice between coordinates of the original point and the reflected point?

$$\left(\frac{1}{2}, -3\frac{1}{2}\right) \quad \left(-\frac{1}{2}, -3\right) \quad (0.25, -0.75)$$

Using number lines to model negative numbers, prove the distance between opposites, and understand the meaning of absolute value easily transfers to the creation and usage of four-quadrant coordinate grids. Points can now be plotted in all four quadrants of a coordinate grid. Differences between numbers can be found by counting the distance between numbers on the grid. Actual computation with negatives and positives is handled in Grade 7.

Instructional Resources/Tools

<http://illuminations.nctm.org/LessonDetail.aspx?ID=L617> In this Illuminations lesson, students learn to plot a point (given its coordinates) and to state the coordinates of a given point. The lesson uses a graphing program for Texas Instruments graphing calculators and also addresses the relationship of ordered pairs that differ only by signs.

<http://mnrussbaum.com/stockshelves/> - This interactive activity allows students the opportunity to work with negative number coordinates and determine their location on the coordinate plane.

<http://www.eduplace.com/math/mathsteps/5/c/index.html> - A useful tutorial to help with the introduction of the coordinate plane.

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