KENTUCKY DEPARTMENT OF EDUCATION

Instructional Supports and Resources

K-PREP Sampler Support
Grade 7

Mathematics

8/20/2012

This document provides teachers with instructional supports for effectively teaching the standards that are measured by the sample released K-PREP mathematics items.

Providing opportunities for students to match situations and sample spaces assists students in visualizing the sample spaces for other situations. Students often struggle when creating organized lists or trees for a situation in order to determine the theoretical probability. Having students start with simpler situations that have fewer elements enables them to have successful experiences with organizing lists and trees diagrams. Ask guiding questions to help students develop methods for creating organized lists and trees for situations with more elements. Creating organized lists, tree diagrams, etc. is not the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models.

Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data. Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same. Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities. Be sure to note examples in simulations when some possibilities are not shown.

Instructional Resources/Tools

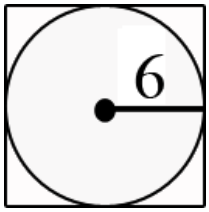
From the National Council of Teachers of Mathematics, Illuminations:
Probability Basics ORC #24 Probability Basics This is a 7+ minute video that explores theoretical and experimental probability with tree diagrams and the fundamental counting principle.

Resources from the Ohio Department of Education
Probability Using Dice ORC #9737 This activity explores the probabilities of rolling various sums with two dice. Extensions of the problem and a complete discussion of the underlying mathematical ideas are included.
How to Fix and Unfair Game ORC #9718 This activity explores a fair game and "How to Fix an Unfair Game."
Remove One Mathline A game is analyzed and the concepts of probability and sample space are discussed. In addition to the lesson plan, the site includes ideas for teacher discussion, extensions of the lesson, additional resources (including a video of the lesson procedures) and a discussion of the mathematical content.
Dart Throwing ORC #10131 The object of this activity is to study an excellent example of the presentation of data. Students analyze an illustration of the 1930 U.S. census compared to the 1960 census to see what inferences they can draw from the data displays.

Activities/Examples:

Experiments can be conducted using various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips. Students can collect data using physical objects or graphing calculator or web-based simulations. Students can also develop models for geometric probability (i.e. a target).
Example:

- If you choose a point in the square, what is the probability that it is not in the circle?



Resources:

Ohio Department of Education. Model Curriculum. March, 2011.
http://www.education.ohio.gov

Arizona Department of Education. Mathematics Resources and Common Core Standards. June, 2011.
http://www.azed.gov/standards-practices/mathematics-standards/

North Carolina State Board of Education. Elementary and Middle Grades Resources.
http://www.ncpublicschools.org/curriculum/mathematics/scos/

Tools for the Common Core Standards. CCSSM Progressions. April, 2011.
http://commoncoretools.me/category/progressions/

Domain	**Geometry**
Cluster	***Draw, construct, and describe geometrical figures and describe the relationships between them.***
Standard	**7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.**
Standards for Mathematical Practice:	MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.

Instructional Strategies

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems.

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps prior to using rulers and protractors to discover and justify the side and angle conditions that will form triangles. Explorations should involve giving students: three side measures, three angle measures, two side measures and an included angle measure, and two angles and an included side measure to determine if a unique triangle, no triangle or an infinite set of triangles results. Through discussion of their exploration results, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. They may realize that for a triangle to result the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees. Students should be able to transfer from these explorations to reviewing measures of three side lengths or three angle measures and determining if they are from a triangle justifying their conclusions with both sketches and reasoning.

Conditions may involve points, line segments, angles, parallelism, congruence, angles, and perpendicularity.

Included in the standards document are critical areas for each grade. **Grade 7 CRITICAL AREA OF FOCUS #3:** Students solve problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

Instructional Resources/Tools

- This cluster lends itself to using dynamic software. Students sometimes can manipulate the software more quickly than do the work manually. However, being able to use a protractor and a straight edge are desirable skills. Protractor, straightedge, ruler, as well as
- Dynamic computer software – such as Geogebra, Wingeom, Geometer's SketchPad.

Activities/Examples:

Is it possible to draw a triangle with a 90˚ angle and one leg that is 4 inches long and one leg that is 3 inches long? If so, draw one. Is there more than one such triangle?

Draw a triangle with angles that are 60 degrees. Is this a unique triangle? Why or why not?
Draw an isosceles triangle with only one 80 degree angle. Is this the only possibility or can you draw another triangle that will also meet these conditions?

Can you draw a triangle with sides that are 13 cm, 5 cm and 6 cm?
Draw a quadrilateral with one set of parallel sides and no right angles.

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Domain	**Ratios and Proportional Relationships**
Cluster	***Analyze proportional relationships and use them to solve real-world and mathematical problems.***
Standards	**7.RP.2 Recognize and represent proportional relationships between quantities.** **a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.** b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.
Standards for Mathematical Practice:	MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

Instructional Strategies

Proportional relationships are developed through the analysis of graphs, tables, equations and diagrams. Ratio tables serve a valuable purpose in the solution of proportional problems. Students in grade 7 need opportunities that allow for a deeper understanding of what a representation of a proportional relationship looks like and what the characteristics are: a straight line through the origin on a graph, a “rule” that applies for all ordered pairs, an equivalent ratio or an expression that describes the situation, etc. Students should not be provided with procedural methods, such as cross-multiplication to solve proportions.

Since percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on “out of 100”; now percents above 100 are encountered and explored.

Providing opportunities to solve problems based within contexts that are relevant to seventh graders will connect meaning to rates, ratios and proportions. Examples include: researching newspaper ads and constructing their own question(s), keeping a log of prices (particularly sales) and determining savings by purchasing items on sale, timing students as they walk a lap on the track and figuring their rates, creating open-ended problem scenarios with and without numbers to give students the opportunity to demonstrate conceptual understanding, inviting students to create a similar question to a given problem and explain their reasoning.

Included in the standards document are critical areas for each grade. **Grade 7 CRITICAL AREA OF FOCUS #1:** Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

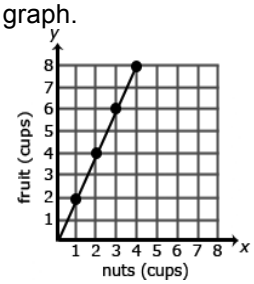
Instructional Resources/Tools

http://mathforum.org/educ475/prop.reasoning3.html online activities and tools to build conceptual understanding
http://www.oercommons.org/browse/alignment/CC.7.RP.2 open educational resources that includes activities and lesson plans
Professional Development Modules to support proportional reasoning
http://www.teachersdomain.org/resource/scl10.plr.module1/
http://www.teachersdomain.org/resource/scl10.plr.module2/
http://www.teachersdomain.org/resource/scl10.plr.module1/
http://www.ket.org/scalecity/ teacher resources and student activities

Activities/Examples:

- A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how you determined the constant of proportionality and how it relates to both the table and graph.

Serving Size	**1**	**2**	**3**	**4**
Cups of Nuts (x)	1	2	3	4
Cups of Fruit (y)	2	4	6	8

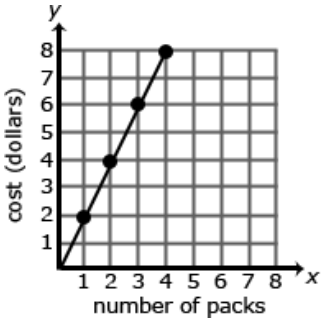


The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1). The constant of proportionality is shown in the first column of the table and by the slope of the line on the graph.

- The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2/pack. Represent the relationship using a table and an equation.

Table:

Number of Packs of Gum (*g*)	Cost in Dollars (*d*)
0	0
1	2
2	4
3	6
4	8



Equation: $2g = d$, where d is the cost in dollars and g is the packs of gum

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as "the number of packs of gum times the cost for each pack is the total cost in dollars". They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost ($g \times 2 = d$).

Additional activity: http://illuminations.nctm.org/LessonDetail.aspx?id=L659

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