KENTUCKY DEPARTMENT OF EDUCATION

Instructional Supports and Resources

K-PREP Sampler Support
Grade 8

Mathematics

8/20/2012

This document provides teachers with instructional supports for effectively teaching the standards that are measured by the sample released K-PREP mathematics items.

Domain	**Statistics and Probability**
Cluster	***Investigate patterns of association in bivariate data.***
Standards	**8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height*.**
Standards for Mathematical Practice:	MP.2. Reason abstractly and quantitatively. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure.
KCAS Connections	8.EE.5; 8.F.3; 8.F.4

Instructional Strategies

Building on the study of statistics using univariate data in Grades 6 and 7, students are now ready to study bivariate data. Students will extend their descriptions and understanding of variation to the graphical displays of bivariate data.

Scatter plots are the most common form of displaying bivariate data in Grade 8. Provide scatter plots and have students practice informally finding the *line of best fit*. Students should create and interpret scatter plots, focusing on outliers, positive or negative association, linearity or curvature. By changing the data slightly, students can have a rich discussion about the effects of the change on the graph. Have students use a graphing calculator to determine a linear regression and discuss how this relates to the graph. Students should informally draw a line of best fit for a scatter plot and informally measure the strength of fit. Discussion should include "What does it mean to be above the line, below the line?"

The study of the line of best fit ties directly to the algebraic study of slope and intercept. Students should interpret the slope and intercept of the line of best fit in the context of the data. Then students can make predictions based on the line of best fit.

Included in the standards document are critical areas for each grade. **Grade 8 CRITICAL AREA OF FOCUS #1**: Students formulate and reason about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and *y*-intercept) in terms of the situation.

Instructional Resources/Tools

This lesson - Glued to the Tube or Hooked to the Books? - provides step-by-step instructions for using the graphing calculator to construct a scatter plot of class data and a line of best fit.

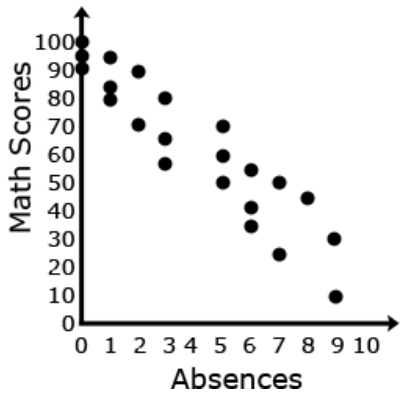
Impact of a Star http://illuminations.nctm.org/LessonDetail.aspx?id=L673 In this activity, students will use the Illuminations Line of Best Fit Activity to plot the data from two teams during the 2004-05 NBA season.

Students model linear data in a variety of settings that range from car repair costs to sports to medicine. Students work to construct scatterplots, interpret data points and trends, and investigate the notion of line of best fit. http://illuminations.nctm.org/LessonDetail.aspx?ID=L298

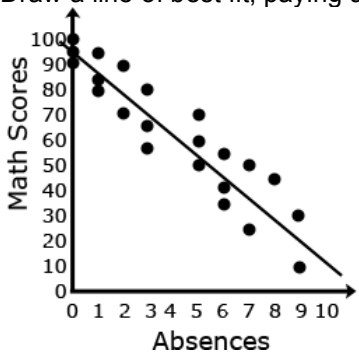
Examples:

- 1. Given data from students' math scores and absences, make a scatterplot.

Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80



- 2. Draw a line of best fit, paying attention to the closeness of the data points on either side of the line.



Resources:

Ohio Department of Education. Model Curriculum. March, 2011.
http://www.education.ohio.gov

Arizona Department of Education. Mathematics Resources and Common Core Standards. June, 2011.
http://www.azed.gov/standards-practices/mathematics-standards/

North Carolina State Board of Education. Elementary and Middle Grades Resources.
http://www.ncpublicschools.org/curriculum/mathematics/scos/

Tools for the Common Core Standards. CCSSM Progressions. April, 2011.
http://commoncoretools.me/category/progressions/

Domain	**Geometry**
Cluster	***Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.***
Standards	**8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve-real world and mathematical problems.**
Standards for Mathematical Practice:	MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.

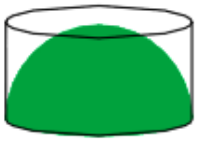
Instructional Strategies

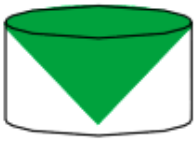
Begin by recalling the formula, and its meaning, for the volume of a right rectangular prism: V = l × w × h. Then ask students to consider how this might be used to make a conjecture about the volume formula for a cylinder:

Most students can be readily led to the understanding that the volume of a right rectangular prism can be thought of as the area of a "base" times the height, and so because the area of the base of a cylinder is πr^2 the volume of a cylinder is $V = \pi r^2 h$.

To motivate the formula for the volume of a cone, use cylinders and cones with the same base and height. Fill the cone with rice or water and pour into the cylinder. Students will discover/experience that 3 cones full are needed to fill the cylinder. This non-mathematical derivation of the formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$, will help most students remember the formula.

In a drawing of a cone inside a cylinder, students might see that that the triangular cross-section of a cone is ½ the rectangular cross-section of the cylinder. Ask them to reason why the volume (three dimensions) turns out to be *less* than ½ the volume of the cylinder. It turns out to be $\frac{1}{3}$.





For the volume of a sphere, it may help to have students visualize a hemisphere "inside" a cylinder with the same height and "base." The radius of the circular base of the cylinder is also the radius of the sphere and the hemisphere. The area of the "base" of the cylinder and the area of the section created by the division of the sphere into a hemisphere is πr^2. The height of the cylinder is also r so the volume of the cylinder is πr^3. Students can see that the volume of the hemisphere is less than the volume of the cylinder and more than half the volume of the cylinder. Illustrating this with concrete materials and rice or water will help students see the relative difference in the volumes. At this point, students can reasonably accept that the volume of the hemisphere of radius r is $\frac{2}{3}\pi r^3$ and therefore volume of a sphere with radius r is twice that or $\frac{4}{3}\pi r^3$. There are several websites with explanations for students who wish to pursue the reasons in more detail. (Note: in the pictures above, the hemisphere and the cone together fill the cylinder.)

Students should experience many types of real-world applications using these formulas. They should be expected to explain and justify their solutions.

Included in the standards document are critical areas for each grade. **Grade 8 CRITICAL AREA OF FOCUS #3:** Students analyze two- and three-dimensional space and figures using distance, angle, similarity, and

congruence, and understanding and applying the Pythagorean Theorem. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Instructional Resources/Tools

#8421 "The Cylinder Problem" - Students build a family of cylinders, all from the same-sized paper, and discover the relationship between the dimensions of the paper and the resulting cylinders. They order the cylinders by their volumes and draw a conclusion about the relationship between a cylinder's dimensions and its volume. http://mathforum.org/brap/wrap2/midlesson.html

National Library of Virtual Manipulatives activity "How High" is an applet that can be used to take an inquiry approach to the formula for volume of a cylinder or cone.

NCTM resources

- Finding Surface Area and Volume
- Blue Cube, 27 Little Cubes (Stella Stunner)
- Volume of a Spheres and Cones (Rich Problem)

A common misconception among middle grade students is that "volume" is a "number" that results from "substituting" other numbers into a formula. For these students there is no recognition that "volume" is a measure – related to the amount of space occupied. If a teacher discovers that students do not have an understanding of volume as a measure of space, it is important to provide opportunities for hands on experiences where students "fill" three dimensional objects. Begin with right- rectangular prisms and fill them with cubes will help students understand why the units for volume are cubed. See Cubes http://illuminations.nctm.org/ActivityDetail.aspx?ID=6

Resources:

Ohio Department of Education. Model Curriculum. March, 2011.
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Domain:	**Functions**
Cluster	***Use functions to model relationships between quantities.***
Standards:	**8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.**
Standards for Mathematical Practice:	MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure.

Instructional Strategies

In Grade 8, students focus on linear equations and functions. Nonlinear functions are used for comparison.

Students will need many opportunities and examples to figure out the meaning of $y = mx + b$. What does m mean? What does b mean? They should be able to "see" m and b in graphs, tables, and formulas or equations, and they need to be able to interpret those values in contexts.

Use graphing calculators and web resources to explore linear and non-linear functions. Provide context as much as possible to build understanding of slope and y-intercept in a graph, especially for those patterns that do not start with an initial value of 0.

Give students opportunities to gather their own data or graphs in contexts they understand. Students need to measure, collect data, graph data, and look for patterns, then generalize and symbolically represent the patterns. They also need opportunities to draw graphs (qualitatively, based upon experience) representing real-life situations with which they are familiar. Probe student thinking by asking them to determine which input values make sense in the problem situations.

Provide students with a function in symbolic form and ask them to create a problem situation in words to match the function. Given a graph, have students create a scenario that would fit the graph. Ask students to sort a set of "cards" to match graphs, tables, equations, and problem situations. Have students explain their reasoning to each other.

From a variety of representations of functions, students should be able to classify and describe the function as linear or non-linear, increasing or decreasing. Provide opportunities for students to share their ideas with other students and create their own examples for their classmates.

Use the slope of the graph and similar triangle arguments to call attention to not just the change in x or y, but also to the rate of change, which is a ratio of the two.

Emphasize key vocabulary. Students should be able to explain what key words mean: e.g., model, interpret, initial value, functional relationship, qualitative, linear, non-linear. Use a "word wall" to help reinforce vocabulary.

Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning—and both types of formulas—are important for developing proficiency with functions.

When input values are not increasing consecutive integers (e.g., when the input values are decreasing, when some integers are skipped, or when some input values are not integers), some students have more difficulty

identifying the pattern and calculating the slope. It is important that all students have experience with such tables, so as to be sure that they do not overgeneralize from the easier examples.

Some students may not pay attention to the scale on a graph, assuming that the scale units are always "one." When making axes for a graph, some students may not using equal intervals to create the scale.
Some students may infer a cause and effect between independent and dependent variables, but this is often not the case.

Some students graph incorrectly because they don't understand that x usually represents the independent variable and y represents the dependent variable. Emphasize that this is a convention that makes it easier to communicate.

Included in the standards document are critical areas for each grade. **Grade 8 CRITICAL AREA OF FOCUS #2:** Students grasp the concept of a function and use functions to describe quantitative relationships. (This does not include function notation.) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

Instructional Resources/Tools

http://illuminations.nctm.org/LessonDetail.aspx?ID=L375 This lesson, from Illuminations, focuses on interpreting and creating graphs that are functions of time. Students focus on graphs of time vs. speed and how many times an event occurred in a specific amount of time.

http://illuminations.nctm.org/LessonDetail.aspx?ID=U188 In this unit there are three Illuminations lessons, where students use movement to reinforce the concepts of linear functions and systems of equations. Multiple representations are used throughout, along with tools such as motion detectors and remote-controlled cars. Students explore how position, speed, and varying motion are reflected in graphs, tables, and algebraic equations.

Graphing calculators – for investigations
Graphing software for computers, including dynamic geometry software
Data-collecting technology, such as motion sensors, thermometers, CBL's, etc.
Graphing applets, such as http://www.mathwolf.com/graph_applets.html

Resources:

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